PHIL 470: Incompleteness and Undecidability

Instructor:	Eric Pacuit (pacuit.org)
Semester:	Spring 2019
Email:	epacuit@umd.edu
Course Website:	myelms.umd.edu/courses/1260203
Office:	Skinner 1103A
Office Hours:	TBA
Class Times:	MW 10:00am - 10:50am
Class Location:	SQH 2120

Course Description

This course will focus on two fundamental theorems of Kurt Gödel: The incompleteness theorems. The first theorem states, roughly, that every formal mathematical theory, provided it is sufficiently expressive and free from contradictions, is incomplete in the sense that there are always statements (in fact, true statements) in the language of the theory which the theory cannot prove. The second incompleteness theorem states that sufficiently expressive arithmetic theories cannot prove their own consistency. We will prove the 1st and 2nd Incompleteness Theorems and survey their technical and philosophical repercussions.

The primary goal of the course is to introduce the technical and philosophical topics that arise when proving Godel's Incompleteness Theorems. Topics to be covered include: formal models of computation (especially elementary recursion theory); Church-Turing Thesis; Gödel's 1st and 2nd incompleteness theorems and their repercussions; Tarski's proof of the undefinability of truth; Undecidability of the Halting Problem; Decidable subsystems of arithmetic; provability logic (Kripke soundness and completeness, arithmetical soundness and completeness, fixed-point theorems); absolute provability; and The Knower Paradox (and epistemic arithmetic).

Prerequisites: PHIL370 (or equivalent logic course), or permission from the instructor.

Literature

The required texts for the course are:

1. The text for this course is from compiled from the Open Logic Project (openlogicproject.org). I compiled two texts for this course. Both texts are available as pdf files on the course website.

    (a) *Sets, Functions, Relations, and First Order Logic* [OL1]: Covers background material on basic set theory, first-order logic and mathematical induction.

    (b) *Incompleteness and Undecidability* [OL2]: Contains the material we will discuss in this course.

I will also assign readings from the following texts (available on the course website):

- P. Smith, *An Introduction to Gödel's Theorems* [IGT], Second Edition, Cambridge, 2013.
- G. Boolos, J. Burgess, and R. Jeffrey, *Computability and Logic* [CL], Cambridge, 2007.
- P. Raatikainen, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/goedel-incompleteness/

The following texts are recommended for additional reading:

- H.-D. Ebbinghaus, J. Flum, and W. Thomas, *Mathematical Logic*, Springer, 1995.
- H. Enderton, *A Mathematical Introduction to Logic*, Academic Press, 2nd Edition, 2001.
- T. Franzen, *Gödel's Theorem: An Incomplete Guide to its Use and Abuse*, A K Peters, 2005.
- M. Fitting, *Incompleteness in the Land of Sets*, Kings College Press, 2007.
- R. Smullyan, *Gödel's Incompleteness Theorems*, Oxford University Press, 1992.

Final Presentation

You will give a presentation on a topic of your choice related to material discussed in the course. We will discuss possible topics later in the semester. Students may want to work in groups to prepare different parts of a topic (e.g., one student could set up and discuss a theorem, then the second student could discuss the proof of the theorem or some interesting application of the theorem). However, *each students must give their own presentation.* The presentation will be no longer than 25 minutes (15-20 minutes to present the material + 5-10 minutes discussion). To receive full credit, you must complete each of the following tasks:

- Prepare a short summary of what you will present. This should not be longer than 1-2 pages, and should include a short paragraph discussing what you will present and list the main reference (or references) you will use for the presentation.
- Prepare slides (either using Keynote, PowerPoint or LaTeX). The presentation **cannot contain more than 5 slides** (with a reasonable amount of text on each slide). You must submit your slides to receive full credit.
- Prepare 3-4 questions that you have or you think that someone else in the class may ask about the material you are presenting. Ideally, the questions should be submitted before you give your presentation.

We will use some class time at the end of the semester for the presentations. In addition, we may need to schedule some time outside of class for the presentations.

Grading Policy

The course requirements are:

- Participation and Attendance (20%): Attendance is required. This is a lecture course and we have a lot of material to cover, so you will easily fall behind if you miss lectures. Please email me if you are going to miss class. Each student receives a maximum of 20 points for attendance. Missing a few classes here and there will not reduce your score by much (Please email me before missing a class). However, more than 3 unexcused absences will lower your score.

- Exam 1 (10%): There will be an exam that contains two parts: an in-class test plus a take-home part. The objective of this exam is to make sure that students are adequately prepared for the course. A review sheet for the in-class part of the exam is available on the course website. This exam will take place early in the semester (the date depends on how much time we spend discussing background material).

- Problem sets (30%): A number of different problems will be assigned throughout the semester (some questions will be asked during lectures and others will be made available on the course website). You should attempt to solve as many of the problems as you can. You can work at your own pace; however, you will need to submit 5 "progress reports" that contain your current answers and/or any questions you have about the problems. Each progress report is worth 10 points and the final problem set (due at the end of the semester) is worth 50 points. You do not necessarily need to solve all the problems. You will be graded on how many problems you attempt and the correctness and completeness of your answers.

  You should think of this as a writing assignment. The goal is to not only come up the correct answer to the questions, but present the answers clearly (indeed, for some problems I may happily provide you with the key idea to the solution). You are encouraged to work in small groups and to discuss your solutions with me. You may discuss the problems with one another or with me as much as you want. *But you must always do the final write-up completely on you own.*

- Final presentation (20%). Each student is required to give a presentation on a topic related to material discussed during the semester (see the section below for details about the final presentation).

- Final exam (20%). The final exam will be an in-class exam given during exam week. A review sheet containing the material that will be covered on the exam will be provided a week or two in advance of the exam.

Tentative Syllabus

Below is a tentative schedule for the semester (consult the course website for more details).

1. Introduction to Incompleteness

   Reading:

   - [OL] Chapter 1
   - [IGT] Chapter 1
   - Introduction to Incompleteness, Smullyan
   - For historical and philosophical context: Kurt Gödel (SEP article), Hilbert's Program (SEP article)

2. Background: Sets, functions, relations, enumerations, first-order logic, language of arithmetic, mathematical induction

   Reading:

   - [OL1] Chapters 1 - 7
   - [CL] Chapters 6.1, 6.2, 12, 13

3. Formal Arithmetic

4. Computability, Primitive Recursive Functions, Recursive Functions

   Reading:

   - [OL2] Chapter 2
   - [CL] Chapters 6.1, 6.2
   - Handouts on primitive recursive functions

5. Arithmetization of syntax (Gödel numbering)

   Reading:

   - [OL2] Chapter 3
   - [IGT] Chapters 19, 20

6. Representability in Q

   Reading:

   - [OL2] Chapter 4
   - [IGT] Chapters 5, 16, 17

7. The First Incompleteness Theorem, Diagonal Lemma

   Reading:

   - [OL2] Sections 5.1 - 5.5

- [IGT] Chapters 21 - 25, 27
- Sep Article on Gödel's Incompleteness Theorem, Section 2

8. The Second Incompleteness Theorem

   Reading:

   - [OL2] Sections 5.6 & 5.7
   - [IGT] Chapters 31, 32

9. Löb's Theorem and Provability Logic

   Reading:

   - [OL2] Section 5.8
   - [CL] Chapter 27

10. Gödel's Disjunction, Absolute Provability and Epistemic Arithmetic, Montague and Kaplan's Knower Paradox (time permitting)

11. Model of arithmetic (time permitting)

12. Second-order arithmetic (time permitting)

Important Dates

- Exam 1: *Tentative date Wed 2/20*
- Spring Break: No class on Monday, March 18 and Wednesday, March 20
- Last Day of Class: Monday, May 13
- Exam Week: May 16 - 22 (the date of the exam will be announced later in the semester when it is scheduled).

Course Policies

The course is subject to the various policies found here:

http://www.ugst.umd.edu/courserelatedpolicies.html

In addition, there is the further e-free policy: please minimize the use of laptops, phones, or other mobile devices during class.