Chapter 5
Focusing on Campaigns

Dominik Klein and Eric Pacuit

All errors in government and in society
are based on philosophic errors which in turn
are derived from errors in natural science.

Marquis de Condorcet

(This quote of Condorcet is taken from the recent book *Gaming the Vote* by William Poundstone [2008, p. 134.])

Abstract One of the important lessons to take away from Rohit Parikh's impressive body of work is that logicians and computer scientists have much to gain by focusing their attention on the intricacies of political campaigns. Drawing on recent work developing a theory of *expressive voting*, we study the dynamics of voters' opinions during an election. In this paper, we develop a model in which the relative importance of the different issues that concern a voter may change either in response to candidates' statements during a campaign or due to unforeseen events. We study how changes in a voter?s attention to the issues influence voting behavior under voting systems such as plurality rule and approval voting. We argue that it can be worthwhile for candidates to reshape public focus, but that doing so can be a complex and risky activity.

Keywords Decision theory · Voting systems · Expressive voting

D. Klein
Tilburg Institute for Logic and the Philosophy of Science, Tilburg, The Netherlands
e-mail: D.Klein@uvt.nl

E. Pacuit (✉)
Department of Philosophy, University of Maryland, College Park, MD, USA
e-mail: epacuit@umd.edu
URL: http://www.pacuit.org

C. Başkent et al. (eds.), *Rohit Parikh on Logic, Language and Society*,
Outstanding Contributions to Logic 11, DOI 10.1007/978-3-319-47843-2_5

5.1 Introduction

Rohit Parikh has noted that political elections are a "bonanza of data" for logicians and game theorists. Much of his recent work (see, especially, his contributions on *social software* Parikh 2002, 2001; Pacuit and Parikh 2006) uses the sophisticated mathematical tools developed by logicians and game theorists to provide penetrating analyses of political phenomena. Even Parikh's more technical work in logic is often motivated by an interest in reasoning about social phenomena (cf. his seminal contribution on game logic Parikh 1985).

In this paper, we follow up on some ideas presented in a recent paper by Dean and Parikh (2011) on the logic of campaigning. They begin their paper with a quote from the satirical newspaper *The Onion*, commenting on the primary race between Barack Obama and Hillary Clinton in 2008:

> After Sen. Barack Obama's comments last week about what he typically eats for dinner were criticized by Sen. Hillary Clinton as being offensive to both herself and the American voters, the number of acceptable phrases presidential candidate can now say is officially down to four. "At the beginning of 2007 there were 38 things candidates could mention in public that wouldn't be considered damming to their campaigns, but now they are mostly limited to 'Thank you all for coming' and 'God bless America"' ABC News chief Washington correspondent George Stephanapoulos said in Sunday's episode of *This Week*.
>
> (The Onion, 2008)

Parikh and Dean develop a formal framework to analyze the phenomenon satirized above.

Suppose that the main issues in an election are represented by a finite set of propositions. Formally, let $\mathcal{L}$ be a propositional language generated from a set At of atomic propositions. For example, the set $\mathsf{At} = \{p, q, r\}$ might represent policies on health care, nuclear energy, and the Israeli-Palestinian conflict. Each voter is assumed to have a preferred ideal world—a complete set of elements of At specifying which policies are enacted. In this setting, *worlds* are most naturally represented as propositional valuation functions $V : \mathsf{At} \to \{-1, 1\}$ where $V(p) = -1$ means that p is false (the policy is not enacted), and $V(p) = 1$ means that p is true (the policy is enacted). A voter i's preference is represented by a function $pref_i : \mathcal{L} \to \{-1, 0, 1\}$ where $pref_i(\varphi) = 1$ means that voter i prefers that φ is true; $pref_i(\varphi) = -1$ means that voter i prefers that φ is false; and $pref_i(\varphi) = 0$ means that voter i is neutral concerning φ. In addition, there is a weighing function for each voter i, $w_i : \mathsf{At} \to [0, 1]$, assigning weights to the different atomic propositions.[1] The weight of a formula $p \in \mathsf{At}$, $w_i(p)$, represents the relative importance of the proposition to voter i. Using these functions, each voter i can assign a value to each world V:

$$Val_i(V) = \sum_{q \in \mathsf{At}} V(q) \times pref_i(q) \times w_i(q).$$

[1]Dean and Parikh normalize the weight function so that $\sum_{p \in \mathsf{At}} w_i(p) = 1$. This simplification is not needed for this paper.

Then, $Val_i(V)$ is a measure of how far way the world V is from the voter's ideal world (which is assigned 1 provided the voters' weights sum to 1).[2]

During a campaign, voters develop theories about the candidates, based on their past utterances (and, perhaps, also on any preconceived ideas the voters may have about the candidates). For a candidate c, let $T_i(c) \subseteq \mathcal{L}$ denote voter i's theory of candidate c. The decision problem that a candidate faces is which statement(s) $\varphi \in \mathcal{L}$ maximizes support among the voters. Solving this decision problem involves two additional features of the voters. First, it depends on how a voter in question will update her theory in response to an utterance. Let $T_i(c) \circ \varphi$ denote i's theory about c that is *updated* with the statement φ. In general, $\circ$ may be any theory change operation, such as an AGM belief revision function (Alchourrón et al. 1985). The second important feature of a voter is how she evaluates theories of candidates. Dean and Parikh consider three different types of voters. Suppose that T is a theory and that $V \models T$ means that $V(\varphi) = 1$ for all $\varphi \in T$. Then, define the following utility functions for a voter i:

$$
\begin{array}{ll}
\text{(pessimistic voter)} & U_i^{min}(T) = \min\{Val_i(V) \mid V \models T\} \\
\text{(optimistic voter)} & U_i^{max}(T) = \max\{Val_i(V) \mid V \models T\} \\
\text{(averaging voter)} & U_i^{ev}(T) = \frac{\sum_{V \models T} Val_i(V)}{|\{V \mid V \models T\}|}
\end{array}
$$

When speaking to a block of voters (i.e., voters who share a theory of the candidate), a candidate is faced with a maximization problem: to choose a statement (which may or may not be consistent with the candidate's actual beliefs) that maximizes the overall utility of the updated theory for a group of voters. Dean and Parikh go on to discuss some intriguing connections with AGM belief revision theory (Alchourrón et al. 1985) (especially, Parikh's important work on splitting languages Parikh 1999).

In this paper, we are interested in studying voters' changing opinions during a campaign. In the model sketched above, the voters' theories of a candidate change in response to that candidate's statements. We take a different perspective in this paper. Instead of allowing voters to change their theory of a candidate during a campaign, we study voters who may *focus* on different issues throughout an election. That is, during a campaign, the relative importance of the different issues for a voter may change either in response to candidates' statements or due to unforeseen events.

The main contribution of this exploratory paper is to raise questions and point out interesting issues rather than to provide a fully worked-out theory. Such a theory will be left for future work. Section 5.2 introduces a framework for reasoning about how voters express themselves when voting. This framework is based on a recent article (Aragones et al. 2011) and has much in common with the Dean and Parikh model sketched above. In Sect. 5.3, we show how to model situations in which the voters' *focus* on issues shifts during an election. Finally, Sect. 5.4 offers some general conclusions and ideas for future work.

---

[2]This model is not only interesting for the theoretician. There are websites, such as www.isidewith.com, that use a variant of this model to rank candidates in upcoming elections according to how close they are to the voter's opinions about a number of relevant policy issues.

5.2 Expressive Voting

Dean and Parikh's model of a campaign is focused on the candidates' decision to make various statements during a campaign and how these statements change the voters' theories of the candidates. In this paper, we conceptualize the dynamics of a campaign differently, by adapting the approach in a paper by Aragones et al. (2011). They develop a model in which a voter's decision to vote is based solely on the need to express herself. To motivate their approach, they cite numerous psychological studies showing that people have an intrinsic need for their opinions to be heard. Of course, as already acknowledged in Aragones et al. (2011), this is an idealization. Voters have many ways to express themselves besides voting. However, for the purposes of this paper, we assume that a voter's only reason for voting is to express her political opinions.[3] Interestingly, this approach drastically changes the analysis of well-studied voting procedures.[4]

In this section, we introduce the model of an election used in Aragones et al. (2011), henceforth called the *AGW* model. Suppose that $T = \{1, \ldots, m\}$ is a set of parties, or candidates. Each party $j \in T$ is characterized by its positions on the various issues of concern $I = \{1, \ldots, n\}$. This is represented as follows: Each $j \in T$ is associated with a vector $\mathbf{p}^j \in [-1, 1]^n$ giving j's positions on each of the issues. The idea is that $p^j_i \in [-1, 1]$ is the degree to which candidate j supports issue i, where 1 denotes total support and -1 total opposition. To simplify the discussion, in this paper, we assume that candidates take extreme positions on each of the issues: for each $i \in I$ and candidate position vector $\mathbf{p} = (p_1, \ldots, p_n)$, we have $p_i \in \{-1, 1\}$. Voters are represented is a similar way. Each voter's opinion about the different issues is represented by a vector $\mathbf{v}^i \in [-1, 1]^n$. For such a vector $\mathbf{v}$, the sign of v_i describes the voter's general attitude towards issue i (i.e., is the voter for or against issue i?), while the absolute value $|v_i|$ reflects the weight that the voter attaches to issue i. The greater the weight, the more important the issue is to the voter. To simplify the notation, we use $\mathbf{p}$ to denote an arbitrary candidate and $\mathbf{v}$ to denote an arbitrary voter.

The key idea behind *expressive voting* is that each ballot in an election is associated with a "statement" giving the amount of support for each issue. To that end, a **ballot** is a vector $\mathbf{x} = (x_1, \ldots, x_m) \in \mathbb{R}^m_+$ (where $\mathbb{R}_+$ is the set of nonnegative real numbers), representing the amount of support for each candidate. For example, **abstention** corresponds to the zero vector (denoted $\mathbf{0}$ where $\mathbf{0}_j = 0$ for each $j = 1, \ldots, m$). The statement made by the ballot $\mathbf{x} = (x_1, \ldots, x_m)$, denoted $s(\mathbf{x})$, is the vector

$$\left(\sum_{j \in T} x_j \times \mathbf{p}^j_1, \ldots, \sum_{j \in T} x_j \times \mathbf{p}^j_n\right) \in \mathbb{R}^n$$

Voter $\mathbf{v}$'s decision problem is to find a ballot that makes a statement as close as possible to her actual position—that is, to find the ballot $\mathbf{x}$ that minimizes the

[3]See, also, Brennan and Lomasky (1993, pp. 40–46) for a discussion of this point.

[4]For instance, the motivation to misrepresent one's position ceases altogether.

Euclidean distance from the statement made by **x** to the voter's own position **v**. More precisely, if F is the set of *feasible* ballots (i.e., the ballots admitted by the voting rule), then voter **v** must solve the following minimization problem:

$$\arg\min_{\mathbf{x}\in F} \text{dist}(\mathbf{v}, s(\mathbf{x})),$$

where $\text{dist}(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_i (x_i - y_i)^2}$ is the usual Euclidean distance. If the solution to this minimization problem is $\mathbf{x} = \mathbf{0} \in \mathbb{R}^m$, then the voter will abstain.[5] The main contribution from Aragones et al. (2011) is a rigorous analysis of the statements that can be made using plurality rule compared to approval voting.

A voting system consists of a set $F \subset \mathbb{R}^m_+$ of *feasible* ballots, together with an aggregation method for selecting a winner from a profile of ballots (one ballot for each voter).

Plurality rule: Each voter selects a single candidate, and the candidate with the most votes is declared the winner. Thus, the feasible ballots are $F^M = \{\mathbf{0}\} \cup \{\mathbf{e}^j\}_{j\leq m}$, where $\mathbf{e}^j$ is the vector with 1 in the jth position and 0 everywhere else.

Example 5.2.1 Suppose that there are three issues $I = \{1, 2, 3\}$ and two candidates $T = \{d, r\}$. Assume that the candidates take extreme opposing positions on the three issues: $\mathbf{p}^d = (1, 1, 1)$ and $\mathbf{p}^r = (-1, -1, -1)$. Consider a voter with the opinion vector $\mathbf{v} = (0, 0.7, -0.1)$. Under plurality rule, there are three possible ballots: $F^M = \{(0, 0), (1, 0), (0, 1)\}$. The statements associated with these ballots are the positions of the two candidates and the null vector:

$$\begin{aligned} s(1, 0) &= 1 * \mathbf{p}^d + 0 * \mathbf{p}^r = (1, 1, 1) \\ s(0, 1) &= 1 * \mathbf{p}^d + 0 * \mathbf{p}^r = (-1, -1, -1) \\ s(0, 0) &= 0 * \mathbf{p}^d + 0 * \mathbf{p}^r = (0, 0, 0) \end{aligned}$$

The voter must choose a ballot $\mathbf{x} \in F^M$ that minimizes $\text{dist}(\mathbf{v}, s(\mathbf{x}))$. The calculations are:

$$\begin{aligned} \text{dist}(\mathbf{v}, s(1, 0)) &= \sqrt{(0-1)^2 + (0.7-1)^2 + (-0.1-1)^2} = \sqrt{2.3} \\ \text{dist}(\mathbf{v}, s(0, 1)) &= \sqrt{(0+1)^2 + (0.7+1)^2 + (-0.1+1)^2} = \sqrt{4.7} \\ \text{dist}(\mathbf{v}, s(0, 0)) &= \sqrt{(0-0)^2 + (0.7-0)^2 + (-0.1-0)^2} = \sqrt{0.5} \end{aligned}$$

Thus, in this case, the voter's best statement is to abstain.

While the plurality rule asks voters to identify the best possible alternative (if one exists), **approval voting** asks voters to identify the candidates that are *approved* (Brams and Fishburn 1983). It turns out that there are two types of ballots that correspond to approval voting in this setting. Following Aragones et al. (2011), the first version defines a ballot by distributing a voter's support among all the candidates that are approved.

[5] Thus, a voter's choice to abstain is due to an inability to express herself in the voting system rather than any cost associated with voting.

Approval Voting, version 1 In approval voting, each voter selects a set of "approved" candidates. In the AGW model, the statement that such a ballot makes is an average over all of the approved candidates' positions. For each $S \subseteq T$, there is a ballot $\mathbf{x}^S$ defined as follows:

$$\mathbf{x}^S = \frac{1}{|S|} \sum_{j \in S} \mathbf{e}^j.$$

So, the total support is divided evenly among the approved candidates. Let F^A denote the set of feasible approval ballots.

Example 5.2.2 Suppose that there are three issues $I = \{1, 2, 3\}$ and three candidates $T = \{l, m, r\}$. Assume that the candidates take the following positions on the three issues: $\mathbf{p}^d = (1, -1, 1)$, $\mathbf{p}^m = (1, 1, 1)$, and $\mathbf{p}^r = (-1, 1, -1)$. Consider a voter with the opinion vector $\mathbf{v} = (0.55, -0.2, 0.8)$. Under plurality rule, there are four possible ballots:

$$F^M = \{(0, 0, 0), (1, 0, 0), (0, 0, 1), (0, 1, 0)\}.$$

As the reader is invited to check, the best statement for the voter in this case is to abstain:

$$\text{dist}(\mathbf{v}, s(0, 0, 0)) = \sqrt{(0.55 - 0)^2 + (-0.2 - 0)^2 + (0.8 - 0)^2} = \sqrt{0.9825}.$$

However, approving of $\{d, m\}$ is preferred to abstaining:

$$\text{dist}(\mathbf{v}, (1, 0, 1)) = \sqrt{(0.55 - 1)^2 + (0 - -0.2)^2 + (1 - 0.8)^2} = \sqrt{0.2825}.$$

According to the above definition, a voter evaluates a coalition of candidates by averaging the positions of its candidates. It is not hard to construct examples in which a voter with moderate positions on the issues may approve of a set of candidates with opposing positions on the same issues. More generally (dropping the assumption that candidates can take only extreme positions on the issues), a moderate voter may prefer a coalition of candidates with extreme opposing positions to a single candidate with relatively moderate positions on the issues. The soundness of the above definition of approval voting relies on voters believing that if the approved candidates are all elected, they will work together to implement more-moderate policies. Of course, examples abound in which the election of officials with opposing positions does not lead to moderate policies, but, rather, to deadlock. This discussion motivates the following variant of approval voting (Klein and Pacuit 2013).

We begin by considering each issue in the election individually. Since we assume that $\mathbf{p}_i \in \{-1, 1\}$ for each candidate $\mathbf{p}$ and issue i, the payoff for a voter $\mathbf{v} = (v_1, \ldots, v_n)$ choosing the candidate $\mathbf{p}$ on issue i is either $|v_i|$ or $-|v_i|$, depending on whether or not the signs of v_i and p_i agree. Formally, we have:

$$val_{\mathbf{v}}(i, \mathbf{p}) = \begin{cases} |v_i| & \text{iff } v_i \cdot p_i \geq 0 \\ -|v_i| & \text{iff } v_i \cdot p_i < 0 \end{cases}$$

Building on this idea, we can precisely define when a voter may approve of a candidate.

Definition 5.2.3 (*k-Approves*) Suppose that $k \in [-1, 1]$ (this is called the **approval coefficient**). A voter $\mathbf{v}$ k-approves of all parties $\mathbf{p}$ that satisfy:

$$\sum p_i v_i \geq k \cdot \sum |v_i|.$$

Since, $\sum p_i v_i$ is the standard scalar product $\mathbf{p} \cdot \mathbf{v}$, and $\sum |v_i|$ is the 1-norm, denoted $|\mathbf{v}|_1$, we have that a voter $\mathbf{v}$ k-approves of a candidate $\mathbf{p}$ provided that:

$$(1) \qquad \mathbf{p} \cdot \mathbf{v} \geq k|\mathbf{v}|_1.$$

Typically, we assume that $k \geq 0$. This means that if a voter k-approves of a candidate, then that voter agrees with the candidate on more issues than she disagrees on.

Definition 5.2.3 has an interesting geometric interpretation. For a vector $\mathbf{x} \in \mathbb{R}^n$ and some angle α, let $\mathcal{C}(x, \alpha)$ be the cone of all vectors $\mathbf{y}$ in $\mathbb{R}^n - \{\mathbf{0}\}$ such that the angle between $\mathbf{x}$ and $\mathbf{y}$ is at most α. Then, we have the following proposition:

Proposition 5.2.4 *Let* $\mathbf{v}$ *be a voter and let* k *be as in the definition of approval voting. Then, there is some angle* α *depending upon* n, k *and* $\mathbf{v}$ *such that for each party* $\mathbf{p}$ *holds*

$$\mathbf{p} \in C(\mathbf{v}, \alpha) \Leftrightarrow \mathbf{p} \cdot \mathbf{v} \geq k|\mathbf{v}|_1.$$

Furthermore, α *satisfies* $\arccos(k) \leq \alpha \leq \arccos(\frac{k}{\sqrt{n}})$.

Proof For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the angle α between $\mathbf{x}$ and $\mathbf{y}$ is described by the following well-known equation:

$$\frac{\mathbf{x} \cdot \mathbf{y}}{|\mathbf{x}|_2 |\mathbf{y}|_2} = \cos \alpha. \tag{5.1}$$

where $|\mathbf{x}|_2 = \sqrt{\sum \mathbf{x}_i^2}$ denotes the Euclidean length of $\mathbf{x}$. On the other hand, inequality (1) can be transformed to

$$\frac{\mathbf{v} \cdot \mathbf{p}}{|\mathbf{v}|_1} \geq k$$

Using a simple algebraic manipulation and multiplying both sides by $\frac{1}{|\mathbf{v}|_2 \sqrt{n}}$, we have

$$\frac{\mathbf{v} \cdot \mathbf{p}}{|\mathbf{v}|_2 \sqrt{n}} \geq \frac{k}{\sqrt{n}} \frac{|\mathbf{v}|_1}{|\mathbf{v}|_2}$$

Since $|\mathbf{p}|_2 = \sqrt{\sum_i 1} = \sqrt{n}$. This is exactly Eq. 5.1 for

$$\alpha = \arccos(\frac{k}{\sqrt{n}} \frac{|\mathbf{v}|_1}{|\mathbf{v}|_2}).$$

The last claim follows from the inequality $|\mathbf{x}|_2 \leq |\mathbf{x}|_1 \leq \sqrt{n}|\mathbf{x}|_2$, for all $\mathbf{x} \in \mathbb{R}^n$. □

Thus, if we interpret the vector $\mathbf{v}$ as providing a voter's general "direction of opinion", the above formula says that a voter k-approves of all parties that lie in roughly the same direction.

We conclude this section by showing that this alternative approach to approval voting is compatible with the definition of plurality rule given above. The following proposition shows that k-approving of a candidate boils down to plurality rule when there are only single-voter ballots. Of course, this depends on the value of the approval coefficient k. We stipulate that a voter $\mathbf{v}$ approves of the candidate $\mathbf{p}$ that minimizes[6] the angle between $\mathbf{v}$ and $\mathbf{p}$. That is, the voter approves of the candidate $\mathbf{p}$ that maximizes the quotient $\frac{\sum v_i \cdot p_i}{\sum |v_i|}$ $\mathbf{v}$.

Proposition 5.2.5 *Suppose that $\overline{T}$ is the set of position vectors for each candidate in T. For any voter $\mathbf{v}$, the voter k-approves of $\mathbf{p}$ with $k = \frac{\sum v_i \cdot p_i}{\sum |v_i|}$ if, and only if,* $\mathrm{dist}(\mathbf{p}^*, \mathbf{v}) = \min_{\mathbf{p} \in \overline{T}} \mathrm{dist}(\mathbf{p}, \mathbf{v})$.

Proof Suppose that $\mathbf{v}$ is a voter for a set of candidates T, and suppose that $\mathbf{p}^*$ is a vector of positions for a candidate. Then, the claim is a direct consequence of the following equivalence:

$$\mathrm{dist}(\mathbf{p}^*, \mathbf{v}) = \min_{\mathbf{p} \in \overline{T}} \mathrm{dist}(\mathbf{p}, \mathbf{v}) \iff \frac{\sum v_i p_i^*}{\sum |p_i^*|} = \max_{p \in P} \frac{\sum v_i p_i}{\sum |p_i|}.$$

Recall that $\mathbf{p}_i \in \{-1, 1\}$ for each topic $i \in I$. Fix a voter $\mathbf{v}$. For any party $\mathbf{p}$, let $U_{\mathbf{p}} \subseteq \{1 \ldots |I|\}$ be defined by:

$$i \in U_{\mathbf{p}} \Leftrightarrow v_i \cdot p_i < 0$$

Thus, $U_{\mathbf{p}}$ is the set of indices where the sign of $\mathbf{v}$ and $\mathbf{p}$ disagree. Now we have:

$$\begin{aligned}
\mathrm{dist}(\mathbf{v}, \mathbf{p}) &= \sqrt{\sum_i (v_i - p_i)^2} \\
&= \sqrt{n + \sum_i v_i^2 - 2\sum_i v_i p_i} \\
&= \sqrt{n + \sum_i v_i^2 - 2\sum_i |v_i| + 4\sum_{i \in U_{\mathbf{p}}} |v_i|}
\end{aligned}$$

Observe that only the last term depends on $\mathbf{p}$. Therefore, we have for any $\mathbf{p}, \mathbf{p}' \in P$:

[6]The voter randomly selects a candidate if there is more than one such candidate.

$$\text{dist}(\mathbf{p}, \mathbf{v}) \leq \text{dist}(\mathbf{p}', \mathbf{v}) \Leftrightarrow \sum_{i \in U_{\mathbf{p}}} |v_i| \leq \sum_{i \in U_{\mathbf{p}'}} |v_i|$$

We also have:

$$\sum_i v_i p_i = \sum_i |v_i| - 2 \sum_{i \in U_{\mathbf{p}}} |v_i|.$$

Thus,

$$\frac{\sum v_i p_i}{\sum |p_i|} \geq \frac{\sum v_i p_i'}{\sum |p_i'|} \Leftrightarrow \sum_{i \in U_{\mathbf{p}}} |v_i| \leq \sum_{i \in U_{\mathbf{p}}'} |v_i|.$$

□

5.3 Focus

During the 2011 German state elections in Baden-Wuerttemberg, it appeared that the governing Christian Conservatives would easily remain in power. The party's position on nuclear energy did not quite match the majority opinion, but most voters were focused on different issues. Then, on March 11, a tsunami hit the Japanese province of Tohoku, causing a major nuclear incident at the Fukushima Daiichi Nuclear Power Plant. Suddenly, nuclear energy was on everyone's mind. This had a drastic effect on the elections: After nearly 40 years of governing, the Christian Conservatives were swept out of office by a Green left coalition (which strongly opposed against nuclear energy).

We can model the German 2011 election scenario using the framework discussed in the previous section. The set of issues is $I = \{i_1, i_2, i_3, i_4\}$ with[7]

i_1 : "We must support the car industry."
i_2 : "We should be conservative about public spending."
i_3 : "We ought to continue nuclear energy."
i_4 : "Do not increase funding for education."

In this framework, the conservatives are represented by a vector $\langle 1, 1, 1, 1 \rangle$, while their two main opponents have a -1 on i_3 and also on some of the other items. Say that the Social Democrats are represented by $\langle 1, -1, -1, -1 \rangle$ and the Greens by $\langle -1, 1, -1, -1 \rangle$.

A typical voter from the Southwest emphasized industry and/or education but displayed only a relatively small concern about nuclear energy. For instance, the following two voters represent a typical voter in this area: $\mathbf{v}_1 = (0.8, 0.9, -0.3, 0.4)$ or

[7] Some people claim that a significant number of voters originally based their decision on a fifth issue i_5: "This party has been in office for the last 40 years". We do not wish to comment on this claim here. We note, however, that our framework is rich enough to incorporate such considerations.

$\mathbf{v}_2 = (0.4, 0.8, -0.3, 0.9)$. Under normal circumstances, this would lead to a crushing victory for the Christian Conservatives, using any of the voting methods discussed in the previous section. However, as stated above, the Fukushima Power Plant incident changed the voters' focus.

Arguably, a change in focus does not necessarily change a voter's general attitude—i.e., the sign of a particular position. It does, however, change the magnitude $|v_i|$ of the entries. Thus, we think of a change in focus as a linear transformation of the space of positions for each voter.[8] This suggests the following definition:

Definition 5.3.1 A **focus matrix** is a diagonal matrix $A \in [0, 1]^{n\times n}$ (i.e., for all $1 \le i, j \le n$, if $i \ne j$, then $A_{ij} = 0$). Voter $\mathbf{v}$'s position after a focus change with A, denoted $\mathbf{v}A$, is calculated in the standard way using matrix multiplication.

The following is a possible focus change matrix triggered by the Fukushima incident:

$$A_{Fuku} = \begin{pmatrix} 0.05 & & & 0 \\ & 0.05 & & \\ & & 1 & \\ 0 & & & 0.05 \end{pmatrix}$$

Clearly, this will make nuclear energy the focus of attention for all voters. After applying this focus change to the two voters mentioned above, the resulting position vectors are $\mathbf{v}_1 A_{Fuku} = (0.04, 0.045, -0.3, 0.02)$ and $\mathbf{v}_2 A_{Fuku} = (0.02, 0.04, -0.3, 0.045)$. Such voters would end up supporting either the Social Democrats or the Green party.

The above example shows that redirecting the voters' focus is a powerful tool that can drastically change the outcome of an election. Indeed, as any political pundit will report, much of the rhetoric during an election is aimed at trying to focus the attention of voters on certain sets of issues. Recall Umberto Eco's famous quote from *Towards a Semiological Guerrilla Warfare* (1967):

> Not long ago, if you wanted to seize political power in a country, you had merely to control the army and the police. Today it is only in the most backward countries that fascist generals, in carrying out a coup d'etat, still use tanks. If a country has reached a high level of industrialization, the whole scene changes. The day after the fall of Khruschev, the editors of Pravda, Izvestiia, the heads of the radio and television were replaced; the army wasn't called out. Today, a country belongs to the person who controls communications.

We conclude this section with a number of examples that illustrate the subtleties involved in changing the focus of a group of voters.

Example 5.3.2 Suppose that there are two candidates $T = \{d, r\}$ competing in a two-topic election (i.e., $I = \{i_1, i_2\}$). The two parties have completely opposing views on

[8] Of course, this is not the only way to represent a change in focus. In general, any transformation (not necessarily linear) on the space of voters' positions can be used to describe a shift of focus during a campaign. A very interesting direction for future research is to explore these different modeling choices.

both topics, say $\mathbf{p}^d = (1, 1)$ and $\mathbf{p}^r = (-1, -1)$. Suppose that almost half of the voters are clearly in favor of the second candidate, $\mathbf{p}^r$. The rest of the voters are relatively undecided, not feeling that either of the parties is particularly close to their views. This example shows that there is a way to focus the voters so that the first candidate, d, is the winner.

To make things more concrete, suppose that there are three voters: $\mathbf{v}_1 = (-1, -0.8)$, $\mathbf{v}_2 = (-1, 0.7)$ and $\mathbf{v}_3 = (1, -0.7 + \epsilon)$. Clearly, d will lose the election given these voters. However, d can win a plurality election by changing the voters' focus using the following matrix:

$$\begin{pmatrix} 0.7 - \delta & 0 \\ 0 & 1 \end{pmatrix}$$

where $\delta \in (0, \epsilon)$. Note that candidate d cannot win the election by focusing on only one of the two issues.

Example 5.3.3 Suppose that there are three candidates $T = \{d, m, r\}$ and six issues $I = \{i_1, \ldots, i_6\}$. Assume that d is in favor of all the topics, $\mathbf{p}^d = (1, 1, 1, 1, 1, 1)$, and r opposes all the topics, $\mathbf{p}^r = (-1, -1, -1, -1, -1, -1)$. The candidates' campaign staffs have determined that d maximizes its share of votes if the voters focus on i_1, i_2 and i_3, while r receives the maximum support when the voters are focused on i_4, i_5 and i_6. In both cases, the maximum support among the voters is enough to win the election using plurality rule. Now, if both candidates d and r think about their public opinion campaigns, then they will try to direct the voters' focus to the issues that maximize their support. However, this may lead to a situation in which candidate m wins a plurality vote.

To fill in the remaining details, suppose that m supports only issues i_3 and i_6 ($\mathbf{p}^m = (-1, -1, 1, -1, -1, 1)$. There are three voters with $\mathbf{v}_1 = \mathbf{v}_2 = (-0.25, 0.3, 1, -0.1, -0.1, -0.1)$ and $\mathbf{v}_3 = (1, -1, 0.9, 1, 1, 1)$. Now, it is not hard to see that:

- In an election in which the voters are focused primarily on i_1, i_2 and i_3, d would win.
- In an election in which the voters are focused primarily on i_4, i_5 and i_6, r would win.
- In an election in which the voters are evenly focused on all the issues $i_1, \ldots, i_6$, m would win. However, if none of the voters focuses on i_1, then d would win the election.

5.4 Concluding Remarks

One of the important lessons to take away from Parikh's impressive body of work is that logicians and computer scientists have much to gain by focusing their attention on the intricacies of political campaigns. Drawing on some recent work developing a theory of *expressive voting*, we have provided some initial observations about the dynamics of voters' opinions during an election. The model of the voters' opinions and candidates' positions on the main issues of a campaign has much in common with Dean and Parikh's model introduced in Sect. 5.1. Our main contributions in this paper are to draw parallels with recent work on expressive voting (Aragones et al. 2011; Gilboa and Vielle 2004) and to stress the importance of the fact that the voters' attention to the main issues may shift during an election.

There are many avenues for future research. Except for some brief remarks when discussing the Dean and Parikh model of campaigns, we did not explicitly make use of any logical machinery. This raises a natural question about the type of logical framework that can naturally capture the phenomena discussed in this paper. We conclude by briefly discussing two additional directions for future work.

Subjective Focus Matrices In this paper, we have assumed that candidates are fully opinionated on the different issues. This modeling choice is an idealization (often, candidates cannot be described as being either fully in favor of or fully against a particular issue). However, the assumption can be justified provided that the issues are suitably fine-grained. There is a trade-off between the size of the set of issues and the richness of the candidates' positions on these issues. Indeed, there is no technical reason preventing us from allowing candidates to adopt positions strictly between -1 and 1 on the issues. Moving to such a model would allow us to represent an aspect of the voter that is present in the Dean and Parikh model but not in our framework.

In Sect. 5.2, we assumed that the candidates' position on each issue is commonly known among the voters. However, as Dean and Parikh note, voters often do not have access to the complete theory of each candidate. This may be due to a lack of information (e.g., the candidate has not stated her full position on the issues) or due to the fact that the voter may not fully trust the candidate (e.g., a candidate's positions represent how likely a voter thinks it is that the candidate will actually follow through on her promises). There are different ways to incorporate this observation into our framework. One approach is to follow Dean and Parikh and assume that each voter has her own theory of the candidates, represented by a set of the candidates' possible position vectors. Alternatively, we could describe the situation in terms of *subjective* focus matrices that depend on both the voters and the candidates.

Generalizing Approval Voting *Range voting* refers to a family of voting systems. The underlying idea behind all of these is that voters are asked to grade candidates using grades which are linearly ordered. The range voting systems differ in the aggregation method used to combine a profile of ballots.[9] Much of the analysis in this paper can be adapted to range voting systems. Suppose that there are n different grades. We can define the analogue of Definition 5.2.3. The n grades are associated with n graded approval coefficients $-1 = t_1 \leq \ldots \leq t_n \in [-1; 1]$. The grade that voter $\mathbf{v}$ assigns to candidate $\mathbf{p}$ is then given by

[9]For instance, majority judgement (Balinski and Laraki 2010) elects the candidate with the highest median grade. Score voting (Smith 2014) elects the candidate with the highest overall mean grade.

$$grade(\mathbf{v}, \mathbf{p}) := \max\{i \mid \mathbf{v} \cdot \mathbf{p} \geq t_i |\mathbf{v}|_1\}.$$

Translated into the language of cones, this means that a voter is associated with a sequence $\mathcal{C}(\mathbf{v}, \alpha_1) \supseteq \ldots \supseteq \mathcal{C}(\mathbf{v}, \alpha_n)$ of narrower and narrower cones, where the first is the entire space. The grade of a party is then given by the highest index i such that $\mathbf{p} \in \mathcal{C}(\mathbf{v}, \alpha_i)$, or equivalently by the number of cones containing $\mathbf{p}$. Approval voting can be seen as a special case of range voting with the grade requirements $t_{disapproval} = -1 \leq t_{approval} = k$.

References

Alchourrón, C. E., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, *50*(2), 510–530.

Aragones, E., Gilboa, I., & Weiss, A. (2011). Making statements and approval voting. *Voting Theory and Decission*, *71*, 461–472.

Balinski, M., & Laraki, R. (2010). *Majority judgment: Measuring, ranking, and electing*. Cambridge: MIT Press.

Brams, S. J., & Fishburn, P. C. (1983). *Approval voting*. Boston: Birkhauser.

Brennan, G., & Lomasky, L. (1993). *Democracy and decision—The pure theory of electoral choice*. Cambridge: Cambridge University Press.

Dean, W., & Parikh, R. (2011). The logic of campaigning. In *Logic and its applications* (pp. 38–49). Berlin: Springer.

Gilboa, I., & Vielle, N. (2004). Majority vote following a debate. *Social Choice and Welfare*, *23*, 115–125.

Klein, D., & Pacuit, E. (2013). Expressive voting: Modeling a voter's decision to vote. Manuscript, 2013.

Pacuit, E., & Parikh, R. (2006). Social interaction, knowledge, and social software. In D. Goldin, S. Smolka, & P. Wegner (Eds.), *Interactive computation* (pp. 441–461). Berlin: Springer.

Parikh, R. (1985). The logic of games and its applications. In M. Karplnski & J. van Leeuwen (Eds.), *Topics in the theory of computation selected papers of the international conference on 'foundations of computation theory'* (Vol. 102, pp. 111–139). Amsterdam: North-Holland.

Parikh, R. (1999). Beliefs, belief revision, and splitting languages. In L. Moss, J. Ginzburg, & M. de Rijke (Eds.), *Logic, language and computation* (Vol. 2, pp. 266–278). Stanford: Center for the Study of Language and Information.

Parikh, R. (2001). Language as social software. In *Future pasts: The analytic tradition in twentieth century philosophy*, pp. 339–350.

Parikh, R. (2002). Social software. *Synthese*, *132*(3), 187–211.

Poundstone, W. (2008). *Gaming the vote*. New York: Hill and Wang.

Smith, W. D. (2014). Rangevoting.org.