Logics of Action, Ability, Knowledge and Obligation

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pacuit.org/esslli2019/epstit

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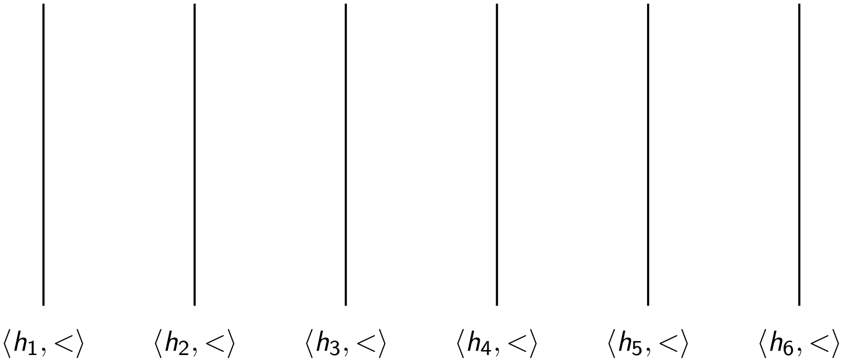
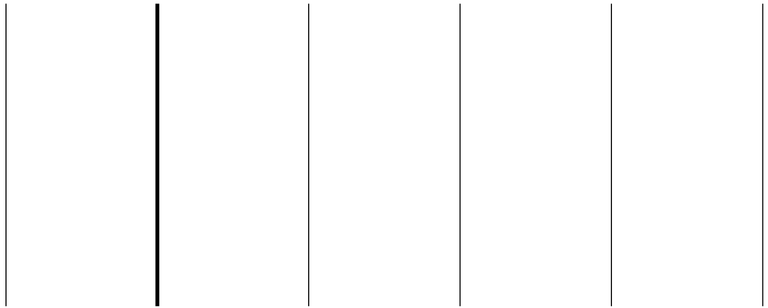
Histories



Each history is a linearly ordered set of moments

Histories



$\langle h_1, < \rangle$ $\langle h_2, < \rangle$ $\langle h_3, < \rangle$ $\langle h_4, < \rangle$ $\langle h_5, < \rangle$ $\langle h_6, < \rangle$

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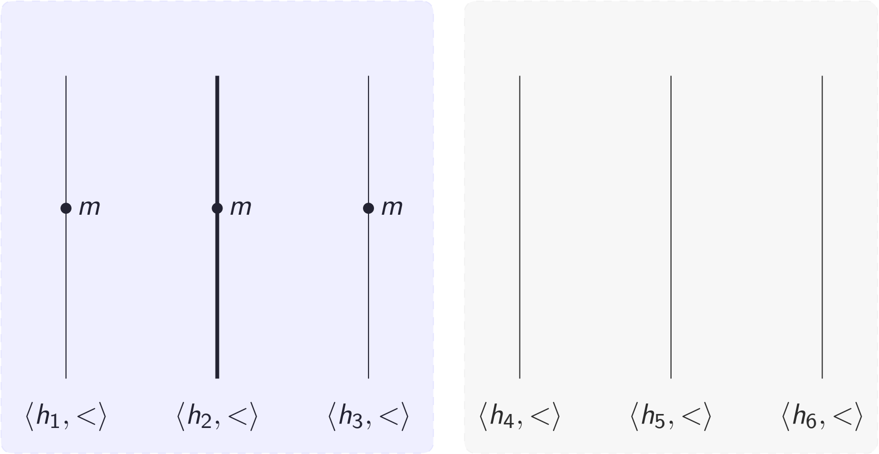
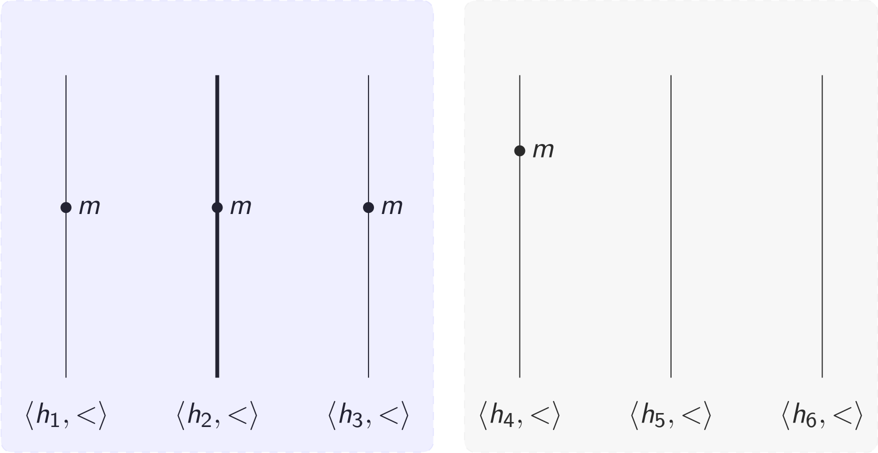
Histories



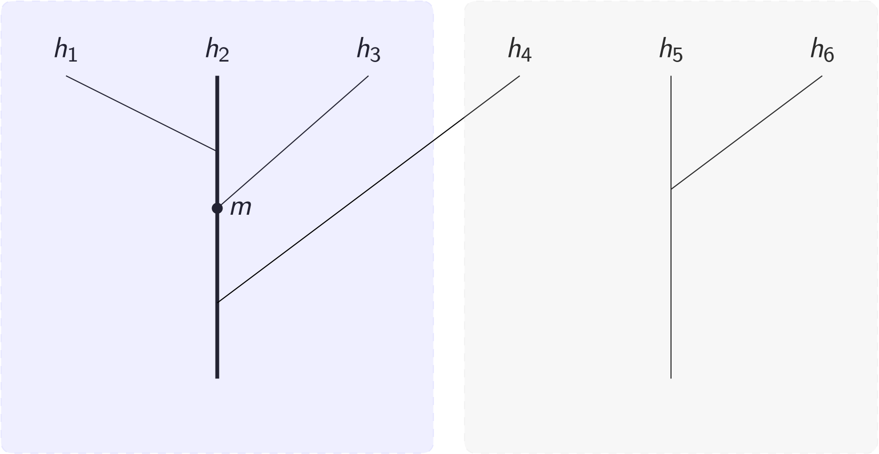
A moment m partitions the set of histories

Histories



h and h' are *equivalent given* m when the initial segments up to m are the *same*

Histories



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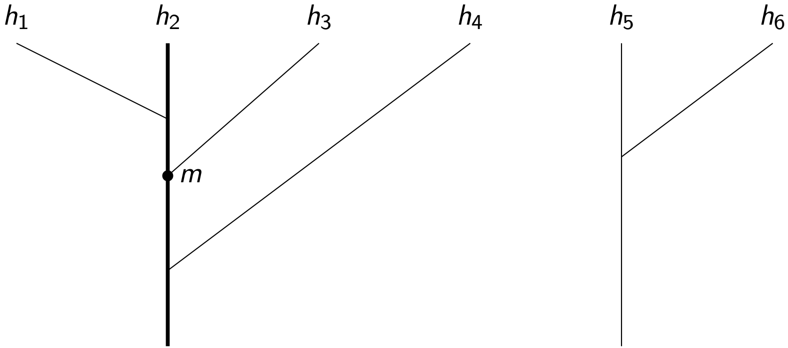
Historic Necessity



The historic necessity modality ($\Box$) quantifies over the histories that are equivalent given a moment m

Historic Necessity

$\mathcal{M}, m/h \models \Box A$ iff $\mathcal{M}, m/h' \models A$ for all $h' \in H^m$

The logic of historic necessity is **S5**:

Prop	propositional tautologies
K	$\Box(A \supset B) \supset (\Box A \supset \Box B)$
T	$\Box A \supset A$
4	$\Box A \supset \Box\Box A$
5	$\neg\Box A \supset \Box\neg\Box A$
Nec	from A infer $\Box A$
MP	from A and $A \supset B$ infer B

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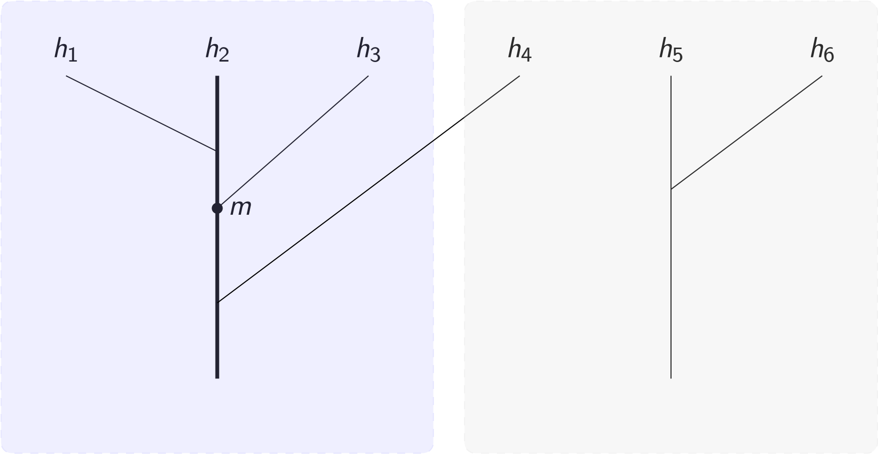
S5 is sound and strongly complete with respect to relational structures in which the relation is an equivalence relation

Choices



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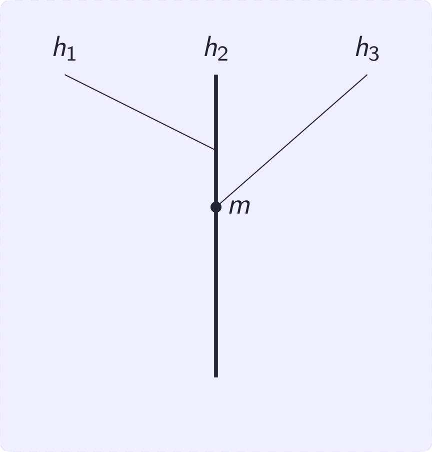
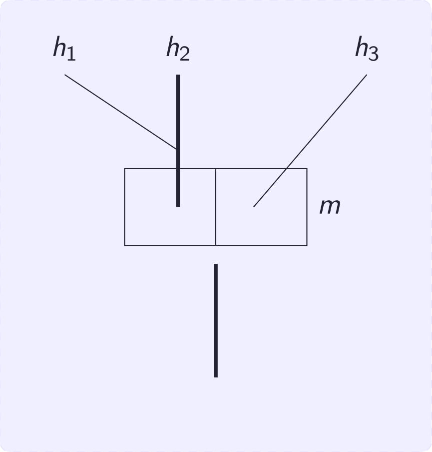


Choices at m is a partition that *refines* the historic necessity partition

Choices



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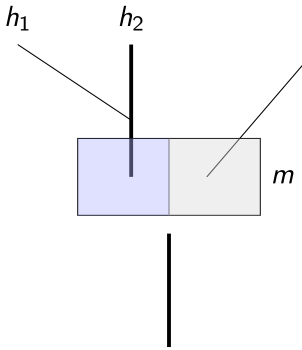

The modality $[\alpha\ cstit : \cdot]$ quantifies over histories in a choice cell

cstit

$\mathcal{M}, m/h \models [\alpha\ cstit\colon A]$ iff $\mathcal{M}, m/h' \models A$ for all $h' \in Choice^m_\alpha(h)$

The logic of *cstit* is **S5**:

Prop	propositional tautologies
K	$[\alpha\ cstit\colon (A \supset B)] \supset ([\alpha\ cstit\colon A] \supset [\alpha\ cstit\colon B])$
T	$[\alpha\ cstit\colon A] \supset A$
4	$[\alpha\ cstit\colon A] \supset [\alpha\ cstit\colon [\alpha\ cstit\colon A]]$
5	$\neg[\alpha\ cstit\colon A] \supset [\alpha\ cstit\colon \neg[\alpha\ cstit\colon A]]$
Nec	from A infer $[\alpha\ cstit\colon A]$
MP	from A and $A \supset B$ infer B

Deliberative stit

$\mathcal{M}, m/h \models [\alpha\ cstit\colon A]$ iff $\mathcal{M}, m/h' \models A$ for all $h' \in Choice^m_\alpha(h)$

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and there is $h'' \in H_m$ such that $\mathcal{M}, m/h'' \not\models A$

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$[\alpha\ dstit\colon A] \leftrightarrow ([\alpha\ cstit\colon A] \wedge \Diamond\neg A)$

$[\alpha\ cstit\colon A] \leftrightarrow ([\alpha\ dstit\colon A] \vee \Box A)$

Historic Necessity and cstit

Since the choice partition *refines* the historic necessity partition, the following is valid

$$\Box A \supset [\alpha \ \mathit{cstit}\colon A]$$

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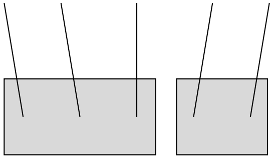
Since the choice partition *refines* the historic necessity partition, the following is valid

$$\Box A \supset [\alpha \textit{ cstit}: A]$$

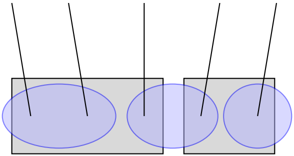
(For all relations R, R', if $R \subseteq R'$, then $[R']A \supset [R]A$ is valid)

Many Agents



Many Agents

Are there logical connections between the *cstit* modality for different agents?



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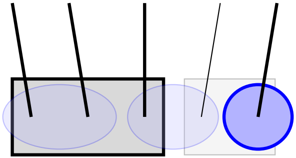

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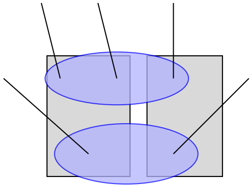

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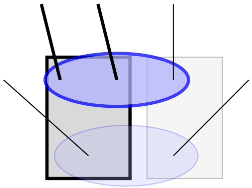

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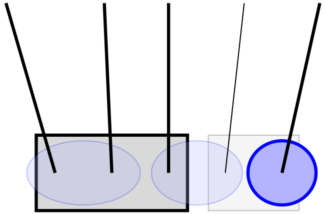

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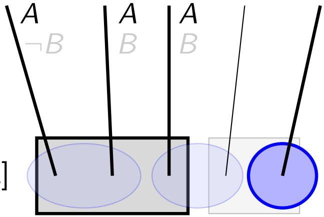
$\Diamond[\alpha \textit{ cstit}: A]$



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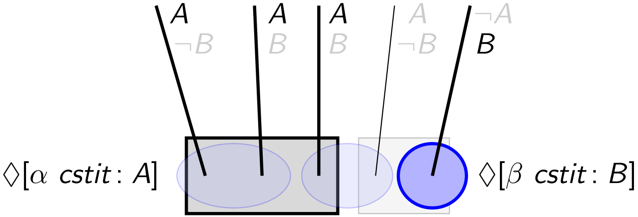

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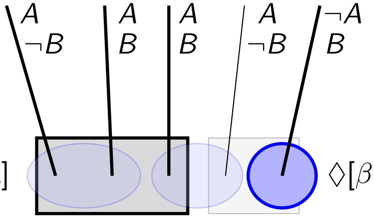


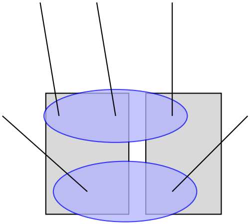
$\Diamond[\beta\ cstit: B]$

$\neg\Diamond([\alpha\ cstit: A] \wedge [\beta\ cstit: B])$

Many Agents

Are there logical connections between the *cstit* modality for different agents?

Independence of agents:
$\bigwedge_\alpha \Diamond[\alpha\ \mathit{cstit}: A_\alpha] \supset \Diamond(\bigwedge_{\alpha \in \mathit{Agt}}[\alpha\ \mathit{cstit}: A_\alpha])$



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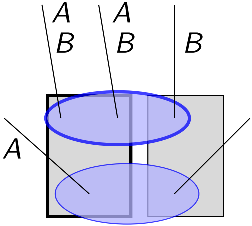

$\Diamond[\alpha \textit{ cstit}: A] \wedge \Diamond[\beta \textit{ cstit}: B]$

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Are there logical connections between the *cstit* modality for different agents?

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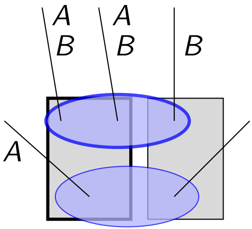
$\bigwedge_\alpha \Diamond[\alpha\ cstit: A_\alpha] \supset \Diamond(\bigwedge_{\alpha\in Agt}[\alpha\ cstit: A_\alpha])$



$\Diamond[\alpha\ cstit: A] \wedge \Diamond[\beta\ cstit: B] \supset \Diamond([\alpha\ cstit: A] \wedge [\beta\ cstit: B])$

Sound and Complete Axiomatization

- **S5** for $\Box$
- **S5** for $[\alpha\ cstit : \cdot]$
- $\Box A \supset [\alpha\ cstit : A]$
- $(\bigwedge_{\alpha \in Agt} \Diamond[\alpha\ cstit : A_\alpha]) \supset \Diamond(\bigwedge_{\alpha \in Agt}[\alpha\ cstit : A_\alpha])$
- Modus Ponens and Necessitation for $\Box$

M. Xu. *Axioms for deliberative STIT*. Journal of Philosophical Logic, Volume 27(5), pp. 505 - 552, 1998.

M. Xu. *On the Basic Logic of STIT with a Single Agent*. Journal of Symbolic Logic, 60(2), pp. 459 - 483, 1995.

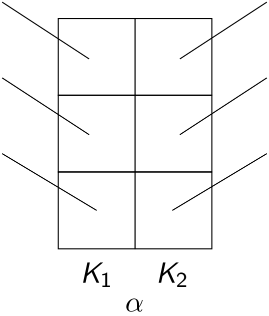
Alternative Axiomatization

P. Balbiani, A. Herzig and N. Troquard. *Alternative axiomatics and complexity of deliberative STIT theories*. Journal of Philosophical Logic, 37:4, 387 - 406, 2008.

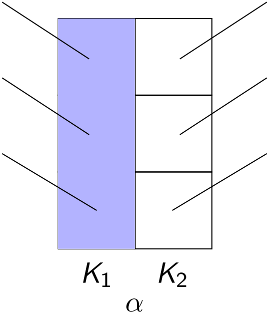
H. Wansing. *Tableaux for multi-agent deliberative-STIT logic*. in Advances in Modal Logic, Volume 6, 503 - 520, 2006.

$$[\alpha \ cstit : [\beta \ cstit : A]]$$

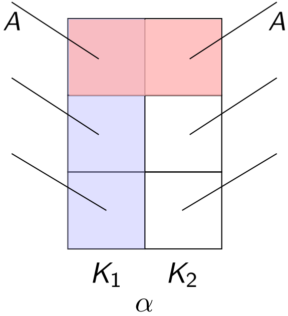
$[\alpha \; cstit: [\beta \; cstit: A]]$



$[\alpha \ \textit{cstit}: [\beta \ \textit{cstit}: A]]$



$[\alpha\ cstit: [\beta\ cstit: A]]$



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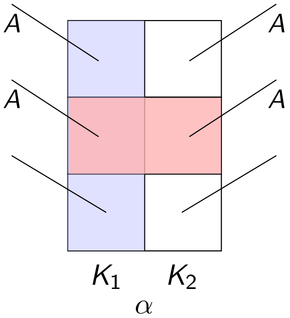
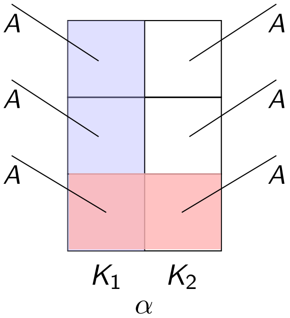
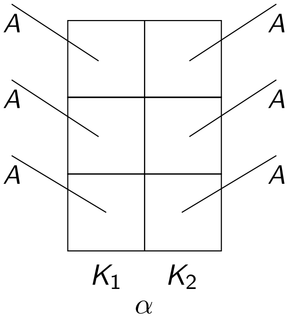


$[\alpha\ cstit\colon [\beta\ cstit\colon A]]$



$$[\alpha \ cstit\colon [\beta \ cstit\colon A]] \equiv \Box A$$



$$\Diamond A \rightarrow \langle \alpha \ cstit\colon \bigwedge_{\beta \in Agt, \beta \neq \alpha} \langle \beta \ cstit\colon A \rangle \rangle$$

where $\langle \alpha \ cstit\colon A \rangle \ \equiv \ \neg[\alpha \ cstit\colon \neg A]$

1. $\Diamond A \supset \langle \alpha \textit{ cstit}: \langle \beta \textit{ cstit}: A \rangle \rangle$
2.
3.
4.
5.
6.
7.
8.
9. $(\Diamond[\beta \textit{ cstit}: B] \wedge \Diamond[\alpha \textit{ cstit}: A]) \supset \Diamond([\beta \textit{ cstit}: B] \wedge [\alpha \textit{ cstit}: A])$

Some (non-)theorems

- $\not\vdash (\Diamond\varphi \wedge \Diamond\psi) \supset \Diamond(\varphi \wedge \psi)$
- $\vdash_{\mathbf{K}} (\Box\varphi \wedge \Box\psi) \supset \Box(\varphi \wedge \psi)$
- $\vdash_{\mathbf{K}} (\Diamond\varphi \wedge \Box\psi) \supset \Diamond(\varphi \wedge \psi)$
- $\vdash_{\mathbf{S5}} (\Diamond\varphi \wedge \Diamond\psi) \supset \Diamond(\Diamond\varphi \wedge \psi)$

1. $\Diamond A \supset \langle \alpha \; cstit\colon \langle \beta \; cstit\colon A \rangle \rangle$
2.
3.
4.
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Some (non-)theorems

- $\vdash_{\mathbf{K}} (\Diamond\varphi \wedge \Box\psi) \supset \Diamond(\varphi \wedge \psi)$
- $\vdash_{\mathbf{K}} (\langle\alpha\ \mathit{cstit}\colon\ \varphi\rangle \wedge [\alpha\ \mathit{cstit}\colon \psi]) \supset \langle\alpha\ \mathit{cstit}\colon (\varphi \wedge \psi)\rangle$

1. $\Diamond A \supset \langle \alpha\ cstit: \langle \beta\ cstit: A\rangle\rangle$
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7.
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Some (non-)theorems

- $\vdash_{\mathbf{S5}} (\Diamond\varphi \wedge \Diamond\psi) \supset \Diamond(\Diamond\varphi \wedge \psi)$

1. $\Diamond A \supset \langle \alpha\ cstit : \langle \beta\ cstit : A \rangle \rangle$
2. $\Diamond[\beta\ cstit : B] \supset \langle \alpha\ cstit : \ \langle \beta\ cstit : [\beta\ cstit : B] \rangle \rangle$
3. $\Diamond[\beta\ cstit : B] \supset \langle \alpha\ cstit : [\beta\ cstit : B] \rangle$
4. $(\Diamond[\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \supset (\langle \alpha\ cstit : [\beta\ cstit : B] \rangle \wedge [\alpha\ cstit : [\alpha\ cstit : A]])$
5. $(\Diamond[\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \supset \langle \alpha\ cstit : ([\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \rangle$
6. $(\Diamond[\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \supset \Diamond([\beta\ cstit : B] \wedge [\alpha\ cstit : A])$
7. $\Diamond(\Diamond[\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \supset \Diamond\Diamond([\beta\ cstit : B] \wedge [\alpha\ cstit : A])$
8. $\Diamond(\Diamond[\beta\ cstit : B] \wedge [\alpha\ cstit : A]) \supset \Diamond([\beta\ cstit : B] \wedge [\alpha\ cstit : A])$
9. $(\Diamond[\beta\ cstit : B] \wedge \Diamond[\alpha\ cstit : A]) \supset \Diamond([\beta\ cstit : B] \wedge [\alpha\ cstit : A])$

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9. $(\Diamond [\beta\ cstit : B] \wedge \Diamond [\alpha\ cstit : A]) \supset \Diamond ([\beta\ cstit : B] \wedge [\alpha\ cstit : A])$

Alternative Axiomatization

- **S5** for $\Box$
- **S5** for $[\alpha\ cstit\colon \cdot]$
- $\Box A \supset [\alpha\ cstit\colon A]$
- $\Diamond A \rightarrow \langle \alpha\ cstit\colon \bigwedge_{\beta \in Agt, \beta \neq \alpha} \langle \beta\ cstit\colon A\rangle\rangle$
- Modus Ponens and Necessitation for $\Box$

P. Balbiani, A. Herzig and N. Troquard. *Alternative axiomatics and complexity of deliberative STIT theories*. Journal of Philosophical Logic, 37:4, 387 - 406, 2008.

Another Axiomatization

- **S5** axioms for $[\alpha \; \mathit{cstit}\colon \cdot]$
- $\Box A \equiv [\alpha \; \mathit{cstit}\colon [\beta \; \mathit{cstit}\colon A]]$
- $\langle \alpha \; \mathit{cstit}\colon \langle \beta \; \mathit{cstit}\colon A \rangle\rangle \supset \langle \gamma \; \mathit{cstit}\colon \bigwedge_{\delta \in \mathcal{A} \setminus \{\gamma\}} \langle \delta \; \mathit{cstit}\colon A \rangle\rangle$ for all $\mathcal{A} \subseteq Agt$
- Modus Ponens and Necessitation for $[\alpha \; \mathit{cstit}\colon \cdot]$

P. Balbiani, A. Herzig and N. Troquard. *Alternative axiomatics and complexity of deliberative STIT theories*. Journal of Philosophical Logic, 37:4, 387 - 406, 2008.

"Flat" Semantics

$\mathcal{M} = \langle W, R, V \rangle$, where

- $W \neq \emptyset$
- R is a function assigning to each $\alpha \in Agt$, $R_\alpha \subseteq W \times W$
- $V : \mathrm{At} \to \wp(W)$.

$\mathcal{M}, w \models [\alpha]A$ iff for all $v \in W$, if $wR_\alpha v$, then $\mathcal{M}, v \models \varphi$.

Generalized Permutation

R satisfies the **general permutation property** iff for all $w, v \in W$ and for all $\alpha, \beta, \gamma \in Agt$, if $(w, v) \in R_\alpha \circ R_\beta$ then there is a $u \in W$ such that $(w, u) \in R_\gamma$ and $(u, v) \in R_\delta$ for all $\delta \in Agt \setminus \{\gamma\}$

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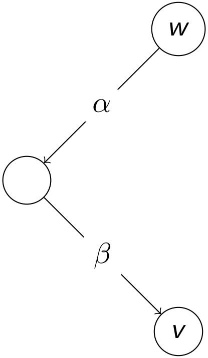

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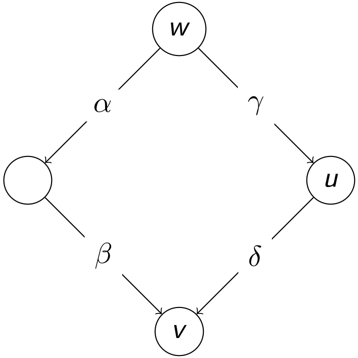

"Flat" Semantics

- The logic of 2-agent stit is the **product** of **S5** (denoted $\mathbf{S5} \oplus \mathbf{S5}$, need to show that the Church-Rosser axiom $\langle\alpha\rangle[\beta]A \supset [\beta]\langle\alpha\rangle A$ is derivable)
- The satisfiability problem for stit with more than two agents is NEXPTIME-complete
- The satisfiability problem for stit with one agent is NP-complete
- The satisfiability problem for stit with more than two agents and group stit modalities is undecidable.

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Other Axiomatizations

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