PHIL 470 - Logical Theory II
Incompleteness and Undecidability

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Semester:	Spring 2022
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Course Website:	umd.instructure.com/courses/1321122
Office:	Skinner 1103A
Office Hours:	Thursdays 2pm - 3pm, Skinner 1103A Fridays 11am - noon, Zoom
Class Times:	TuTh 12:30pm - 1:45pm
Class Location:	MMH 1304

Course Description

This course will focus on Kurt Gödel's first and second incompleteness theorems. The first incompleteness theorem states, roughly, that every logical system that is sufficiently expressive and free from contradictions is incomplete in the sense that there are always statements such that neither the statement nor its negation can be proved. The second incompleteness theorem states that sufficiently expressive arithmetic theories cannot prove their own consistency. We will prove the 1st and 2nd Incompleteness Theorems and survey their technical and philosophical repercussions.

The primary goal of the course is to introduce the technical and philosophical topics that arise when proving Gödel's Incompleteness Theorems. Topics to be covered include: formal models of computation (especially elementary recursion theory); the Church-Turing Thesis; Gödel's 1st and 2nd incompleteness theorems and their repercussions; Tarski's proof of the undefinability of truth; Undecidability of the Halting Problem; provability logic - Kripke soundness and completeness, arithmetical soundness and completeness, fixed-point theorems (time permitting); and absolute provability and The Knower Paradox (time permitting); and non-standard models of arithmetic (time permitting).

Prerequisites: PHIL370 (or equivalent logic course), or permission from the instructor.

Literature

The readings for the course will be made available on the course website. Readings will be taken from the following texts:

1. *Incompleteness and Undecidability.* This text is compiled from the Open Logic Project: (openlogicproject.org).
2. G. Boolos, J. Burgess, and R. Jeffrey, *Computability and Logic*, Cambridge, 2007.
3. J. D. Hamkins, *Lectures on the Philosophy of Mathematics*, the MIT press, 2021.

4. P. Smith, *An Introduction to Gödel's Theorems*, Second Edition, Cambridge, 2020. `https://www.logicmatters.net/resources/pdfs/godelbook/GodelBookLM.pdf`.

5. P. Raatikainen, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy, https://plato.stanford.edu/entries/goedel-incompleteness/.

The following texts are recommended for additional reading:

- I. Chiswell and W. Hodges, *Mathematical Logic*, Oxford University Press, 2007.
- H.-D. Ebbinghaus, J. Flum, and W. Thomas, *Mathematical Logic*, Springer, 1995.
- H. Enderton, *A Mathematical Introduction to Logic*, Academic Press, 2nd Edition, 2001.
- M. Fitting, *Incompleteness in the Land of Sets*, Kings College Press, 2007.
- T. Franzen, *Gödel's Theorem: An Incomplete Guide to its Use and Abuse*, A K Peters, 2005.
- R. Smullyan, *Gödel's Incompleteness Theorems*, Oxford University Press, 1992.

Final Presentation

You will give a presentation on a topic of your choice related to material discussed in the course. We will discuss possible topics later in the semester. Students may want to work in groups to prepare different parts of a topic (e.g., one student could set up and discuss a theorem, then the second student could discuss the proof of the theorem or some interesting application of the theorem). However, *each students must give their own presentation.* The presentation will be no longer than 25 minutes (15-20 minutes to present the material + 5-10 minutes discussion). To receive full credit, you must complete each of the following tasks:

- Prepare a short summary of what you will present. This should not be longer than 1-2 pages, and should include a short paragraph discussing what you will present and list the main reference (or references) you will use for the presentation.
- Prepare slides (either using Keynote, PowerPoint or LaTeX). The presentation **cannot contain more than 5 slides** (with a reasonable amount of text on each slide). You must submit your slides to receive full credit.
- Prepare 2-3 questions that you have or you think that someone else in the class may ask about the material you are presenting. Ideally, the questions should be submitted before you give your presentation.

We will use some class time at the end of the semester for the presentations. In addition, we may need to schedule some time outside of class for the presentations.

Grading Policy

The course requirements are:

- Participation (40%): There will be a number of "tutorials" assigned during the semester. You will complete these questions either in-class or before class, and we will discuss the answers in class. You will be graded on completion, and will have a chance to make up any answers that you missed.

  This is a lecture course and we have a lot of material to cover, so you will easily fall behind if you miss lectures. Please email me if you are going to miss class.

- Problem sets (30%): A number of different problems will be assigned throughout the semester (some questions will be asked during lectures and others will be made available on the course website). You should attempt to solve as many of the problems as you can. You can work at your own pace; however, you will need to submit "progress reports" that contain your current answers and/or any questions you have about the problems. You do not necessarily need to solve all the problems. You will be graded on how many problems you attempt and the correctness and completeness of your answers.

  You should think of this as a writing assignment. The goal is to not only come up the correct answer to the questions, but present the answers clearly (indeed, for some problems I may happily provide you with the key idea to the solution). You are encouraged to work in small groups and to discuss your solutions with me. You may discuss the problems with one another or with me as much as you want. *But you must always do the final write-up completely on you own.*

- Final presentation (20%). Each student is required to give a presentation on a topic related to material discussed during the semester (see the section below for details about the final presentation).

- Final exam (10%). The final exam will be an in-class exam given during exam week. A review sheet containing the material that will be covered on the exam will be provided a week or two in advance of the exam.

Tentative Syllabus

Below is a tentative schedule for the semester (consult the course website for more details).

1. Introduction to Incompleteness (1 week)
2. Background: Sets, functions, relations, enumerations, mathematical induction first-order logic, language of arithmetic, natural deduction (1 week)
3. Formal Arithmetic (1 week)
4. Computability, Primitive Recursive Functions, Recursive Functions (1 - 2 weeks)
5. Arithmetization of syntax (Gödel numbering) (1 week)

6. Representability in Q (2 weeks)
7. The Diagonal Lemma and the First Incompleteness Theorem (1 week)
8. The Second Incompleteness Theorem (1 week)
9. Derivability conditions and Löb's Theorem (1-2 weeks)

Time permitting we will cover some of the following topics (depending on student interest).

1. Provability Logic
2. Gödel's Disjunction, Absolute Provability and Epistemic Arithmetic, Montague and Kaplan's Knower Paradox
3. Models of arithmetic
4. Second-order arithmetic

Important Dates

- Spring Break: March 20 - 27, No class on Tuesday, March 22 and Thursday, March 24
- Last Day of Class: Tuesday May 10
- Exam Week: May 12 - 18 (the date of the exam will be announced later in the semester when it is scheduled).

Learning Outcomes

Students who successfully complete this course will be prepared to:

- Define the logical systems and main concepts (including mathematical models of computation) used in the proof of the incompleteness theorems
- Reproduce an outline of the proof of Gödel's Incompleteness Theorems and related results
- Explain the philosophical implications of Gödel's Theorems and related results
- Discuss the Church-Turing thesis and the undecidability phenomenon

Course Policies

The course is subject to the various policies found here:

http://www.ugst.umd.edu/courserelatedpolicies.html

In addition, there is the further e-free policy: please minimize the use of laptops, phones, or other mobile devices during class.