Tools for Formal Epistemology: Doxastic Logic, Probability and Default Logic

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Lecture 2

ESSLLI 2023

Preface Paradox

D. Makinson. *The Paradox of the Preface*. Analysis, 25, 205 - 207, 1965.

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$B_A(\neg(s_1 \wedge s_2 \wedge \cdots \wedge s_n))$

But $\{s_1, \ldots, s_n, \neg(s_1 \wedge \cdots \wedge s_n)\}$ is logically inconsistent.

Preface Paradox

A philosopher who asserts "all of my present philosophical positions are correct" would be regarded as rash and over-confident

A philosopher who asserts "at least some of my present philosophical beliefs will turn out to be incorrect" is simply being sensible and honest.

Preface Paradox

1. each belief from the set $\{s_1, \ldots, s_n, s_{n+1}\}$ is rational
2. the set $\{s_1, \ldots, s_n, s_{n+1}\}$ of beliefs is rational.

1. does not necessarily imply 2.

Preface Paradox: The Problem

"The author of the book is being rational even though inconsistent. More than this: he is being rational even though he believes each of a certain collection of statements, which *he knows* are logically incompatible....this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold incompatible beliefs."

D. Makinson. *The Paradox of the Preface*. Analysis, 25, 205 - 207, 1965.

Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

G. Wheeler. *A Review of the Lottery Paradox*. Probability and Inference: Essays in honor of Henry E. Kyburg, Jr., College Publications, 2007.

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"Surely if a sheer probability is ever sufficient to warrant the acceptance of a hypothesis, this is a case"

For each lottery ticket t_i ($i = 1, \ldots, 1000000$), the agent believes that t_i will loose $B_A(\neg$'t_i will win')

Lottery Paradox

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So, the conjunction $\bigwedge_{i=1}^{1000000}$ 't_i will not win' should be accepted. That is, the agent should rationally accept that *no lottery ticket will win*.

But, this is a fair lottery, so at least one ticket is *guaranteed* to win!

The Lottery Paradox

Kyburg: The following are inconsistent,

1. It is rational to accept a proposition that is very likely true,
2. It is not rational to accept a propositional that you are aware is inconsistent
3. It is rational to accept a proposition P and it is rational to accept another proposition P' then it is rational to accept $P \wedge P'$

Conceptions of Belief

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Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

Conceptions of Beliefs: Questions

What are the *formal constraints* on rational belief?

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What are the *formal constraints* on rational belief?

- rational graded beliefs should obey the laws of probability
- rational all-out beliefs should be consistent/deductively closed
- how should we justify these constraints?

D. Christensen. *Putting Logic in its Place*. Oxford University Press.

Suppose that W is a set of states (the *set of outcomes*).

A σ-algebra is a set $\Sigma \subseteq \wp(W)$ such that

- $W \in \Sigma$
- If $A \in \Sigma$, then $\overline{A} \in \Sigma$
- If $\{A_i\}$ is a countable collection of sets from Σ, then $\bigcup_i A_i \in \Sigma$

A **probability function** is a function $p : \Sigma \to [0,1]$ satisfying:

- $p(W) = 1$
- $p(A \cup B) = p(A) + p(B)$ whenever $A \cap B = \emptyset$

(W, Σ, p) is called a probability space.

Probability

Kolmogorov Axioms:

1. For each E, $0 \leq p(E) \leq 1$
2. $p(W) = 1$, $p(\emptyset) = 0$
3. If $E_1, \ldots, E_n, \ldots$ are pairwise disjoint ($E_i \cap E_j = \emptyset$ for $i \neq j$), then $p(\bigcup_i E_i) = \sum_i p(E_i)$

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- $p(\overline{E}) = 1 - p(E)$ ($\overline{E}$ is the complement of E)
- If $E \subseteq F$ then $p(E) \leq p(F)$
- $p(E \cup F) = p(E) + p(F) - p(E \cap F)$

Suppose that $(\mathcal{L}, \models)$ is a logic. A probability function is a map $p : \mathcal{L} \to [0, 1]$ such that

1. For each E, $0 \leq p(\varphi) \leq 1$
2. $p(\varphi) = 1$ if $\models \varphi$
3. If $p(\varphi \vee \psi) = p(\varphi) + p(\psi)$ when $\models \neg(\varphi \wedge \psi)$.

I.J. Good. *46,656 Varieties of Bayesians*. Good Thinking: The Foundations of Probability and Its Applications, University of Minnesota Press (1983).

Conditional Probability

The probability of E *given* F, dented $p(E|F)$, is defined to be

$$p(E|F) = \frac{p(E \cap F)}{p(F)}.$$

provided $P(F) > 0$.

Bridge Principles

Probability 1: $B(A)$ iff $P(A) = 1$

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Decision-theoretic accounts: $B(A)$ iff $\sum_{w \in W} P(\{w\}) \cdot u(B(A), w)$ has such-and-such property

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The Nihilistic proposal: "...no explication of belief is possible within the confines of the probability model."

The Review Paradox

H. Leitgeb. *The Review Paradox: On the Diachronic Costs of Not Closing Rational Belief Under Conjunction*. Nous, 2013.

B_t is the set of propositions believed at time t

P_t is the agent's degree of belief function at time t

$t' > t$

P1 If the degrees of belief that the agents assigns to two propositions are identical then either the agent believes both of them or neither of them.

For all X, Y: if $P_t(X) = P_t(Y)$, then $B_t(X)$ iff $B_t(Y)$.

P2 If the agent already believes X, then updating on the piece of evidence X does not change her system of (all-or-nothing) beliefs at all.

For all X: if the evidence that the agent obtains between t and $t' > t$ is the proposition X, but it holds already that $B_t(X)$, then for all Y:

$$B_{t'}(Y) \text{ iff } B_t(Y)$$

P3 When the agent learns, this is captured probabilistically by conditionalization.

For all X (with $P_t(X) > 0$): if the evidence that the agent obtains between t and $t' > t$ is the proposition X, but it holds already that $B_t(X)$, then for all Y:

$$P_{t'}(Y) = P_t(Y \mid X)$$

Assume $B_t(A), B_t(B)$ but not $B_t(A \cap B)$

- Suppose that the agent receive A as evidence.
- $P_{t'}(B) = P_t(B \mid A) = P_t(A \cap B \mid A) = P_{t'}(A \cap B)$.
- By P1, the agent must have the same doxastic attitude towards B and $A \cap B$.
- By P2, the agent's attitude towards B and $A \cap B$ must be the same at t' as at t.
- But, $B_t(B)$ and not $B_t(A \cap B)$

$B_t(A), B_t(B)$

$\neg B_t(A \cap B)$

$0 < P_t(A) < 1$

t t'

Assumption

$B_t(A), B_t(B)$

$\neg B_t(A \cap B)$

$0 < P_t(A) < 1$

$P_{t'}(B) = P_t(B \mid A)$

$P_{t'}(A \cap B) = P_t(A \cap B \mid A) = P_t(B \mid A)$

t

Receives evidence A

t'

By P3

$B_t(A), B_t(B)$

$\neg B_t(A \cap B)$

$0 < P_t(A) < 1$

$P_{t'}(B) = P_t(B \mid A)$

$P_{t'}(A \cap B) = P_t(A \cap B \mid A) = P_t(B \mid A)$

$B_{t'}(B)$ iff $B_{t'}(A \cap B)$

t

Receives evidence A

t'

By P1

$B_t(A), B_t(B)$

$\neg B_t(A \cap B)$

$0 < P_t(A) < 1$

$P_{t'}(B) = P_t(B \mid A)$

$P_{t'}(A \cap B) = P_t(A \cap B \mid A) = P_t(B \mid A)$

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t Receives evidence A t'

$B_t(A)$ iff $B_{t'}(A)$

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$B_t(A \cap B)$ iff $B_{t'}(A \cap B)$

By P2

$B_t(B)$

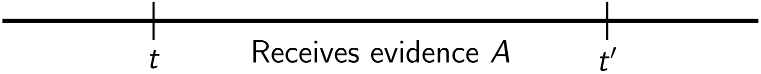
$\neg B_t(A \cap B)$

$0 < P_t(A) < 1$

$P_{t'}(B) = P_t(B \mid A)$

$P_{t'}(A \cap B) = P_t(A \cap B \mid A) = P_t(B \mid A)$

$B_{t'}(B)$ iff $B_{t'}(A \cap B)$



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Assume with the paradox that the author believes each of $T_1, \ldots, T_n$ without believing $T_1 \cap \cdots T_n$.

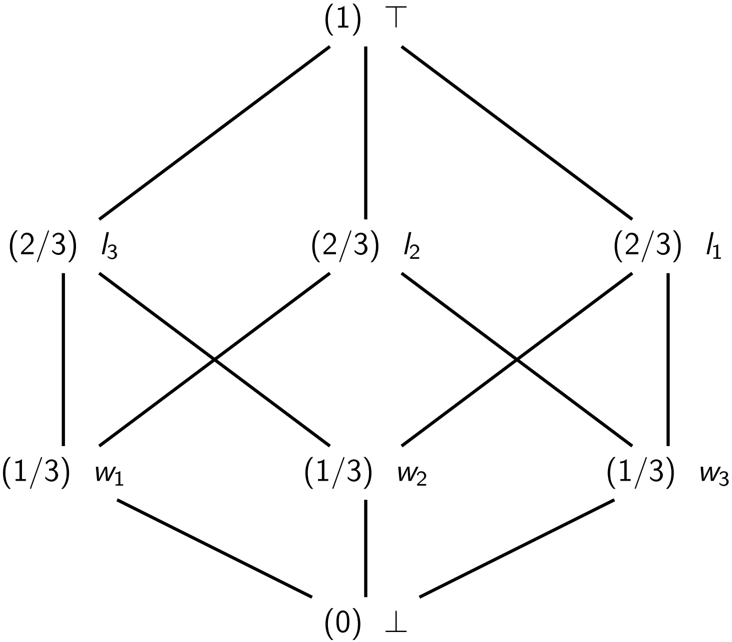
Let m be the maximal number less than n so that the author believes $T_1 \cap \cdots \cap T_m$ without believing $T_1 \cap \cdots T_{m+1}$; clearly, there must be such a number m in the preface paradox situation.

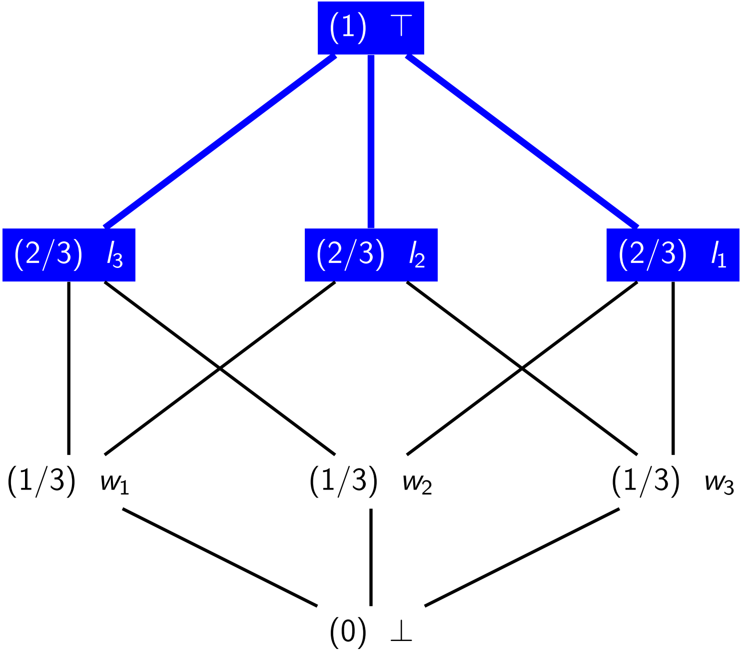
Suppose that someone writes a review of the author's book in which the reviewer strengthens the author's case for $T_1 \cap \cdots \cap T_m$, without saying anything at all about T_{m+1} or any other of the author's theses (maybe the reviewer is simply not interested in them): "What I can say about this book is that $T_1 \cap \cdots \cap T_m$ definitely is the case."

Assume that the author is rationally absorbing this report—updating on the proposition $T_1 \cap \cdots \cap T_m$ if stated in qualitative terms, and, if stated in quantitative terms, updating on $T_1 \cap \cdots \cap T_m$ by conditionalization: then given $P1 - P3$, one encounters a contradiction: It seems that the author cannot rationally take in a perfectly positive review of her book. Call this the review paradox.

Beliefs that obey the Lockean thesis can be undermined by new evidence that is consistent with the agent's current beliefs.

For each $i = 1, 2, 3$, let l_i be the proposition Ticket i won't win (and w_i is the proposition that "ticket i will win"). And let us set our threshold for Lockean belief at $r = 0.6$.


(1) ⊤
(2/3) l_3
(2/3) l_2
(2/3) l_1
(1/3) w_1
(1/3) w_2
(1/3) w_3
(0) ⊥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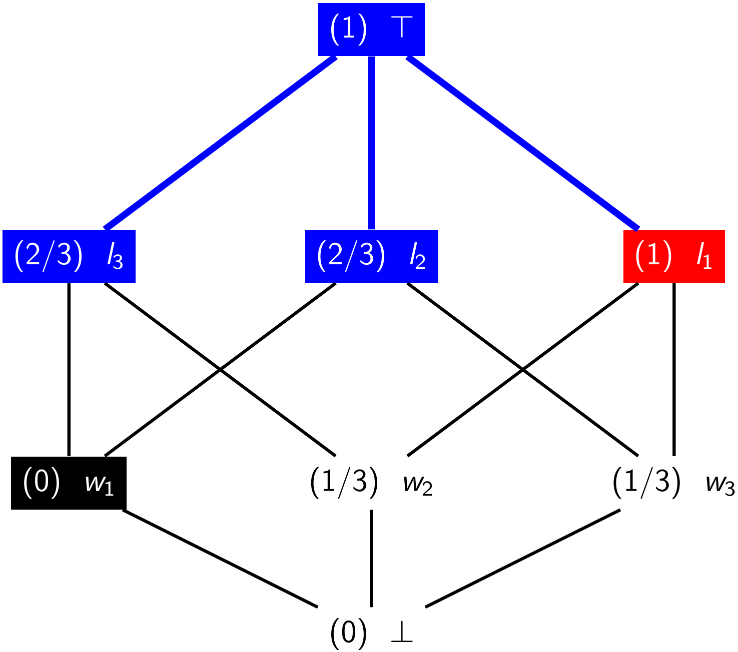

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(2/3) l_2
(1) l_1
(0) w_1
(1/3) w_2
(1/3) w_3
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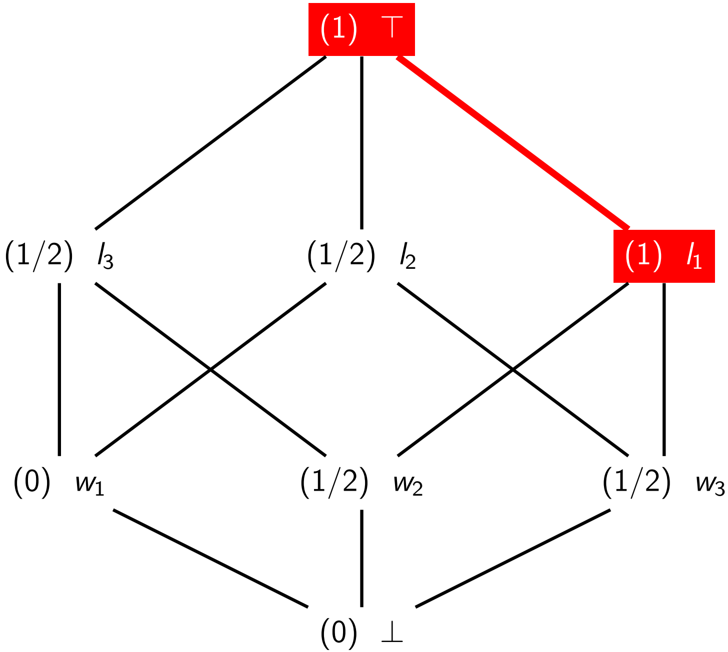
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(1/2) l_2
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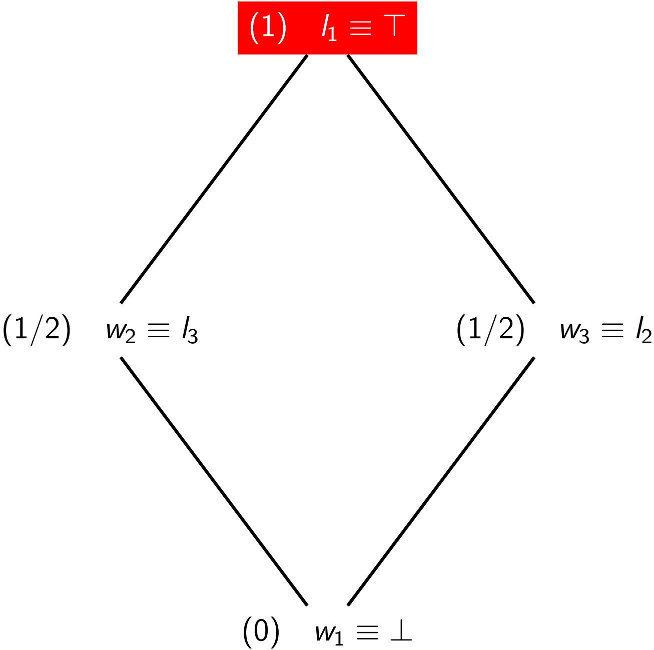
(0) w_1
(1/2) w_2
(1/2) w_3
(0) ⊥


(1) $l_1 \equiv \top$
(1/2) $w_2 \equiv l_3$
(1/2) $w_3 \equiv l_2$
(0) $w_1 \equiv \bot$

Resiliency, Robust Belief, Stable Belief

B. Skyrms. *Resiliency, propensities, and causal necessity*. Journal of Philosophy, 74:11, pgs. 704 - 713, 1977.

A. Baltag and S. Smets. *Probabilistic Belief Revision*. Synthese, 2008.

H. Leitgeb. *Reducing belief simpliciter to degrees of belief*. Annals of Pure and Applied Logic, 16:4, pgs. 1338 - 1380, 2013.

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Probability

Let W be a set of states and $\mathfrak{A}$ a σ-algebra: $\mathfrak{A} \subseteq \wp(W)$ such that

- $W, \emptyset \in \mathfrak{A}$
- if $X \in \mathfrak{A}$ then $W - X \in \mathfrak{A}$
- if $X, Y \in \mathfrak{A}$ then $X \cup Y \in \mathfrak{A}$
- if $X_0, X_1, \ldots \in \mathfrak{A}$ then $\bigcup_{i \in \mathbb{N}} X_i \in \mathfrak{A}$.

Probability

$P : \mathfrak{A} \to [0,1]$ satisfying the usual constraints

- $P(W) = 1$
- (finite additivity) If $X_1, X_2 \in \mathfrak{A}$ are pairwise disjoint, then $P(X_1 \cup X_2) = P(X_1) + P(X_2)$

$P(Y|X) = \frac{P(Y \cap X)}{P(X)}$ whenever $P(X) > 0$. So, $P(Y|W)$ is $P(Y)$.

- P is countably additive (σ-additive): if $X_1, X_2, \ldots, X_n, \ldots$ are pairwise disjoint members of $\mathfrak{A}$, then $P(\bigcup_{n \in \mathbb{N}} X_n) = \sum_{n \in \mathbb{N}} P(X_n)$

P-stability^r

Definition. Let P be a probability measure on $\mathfrak{A}$ over W, let $0 \leq t < 1$. For all $X \in \mathfrak{A}$:

$$X \text{ is } P\text{-stable}^t \text{ if and only if for all } Y \in \mathfrak{A} \text{ with } Y \cap X \neq \emptyset \text{ and } P(Y) > 0\text{:}$$
$$P(X|Y) > t.$$

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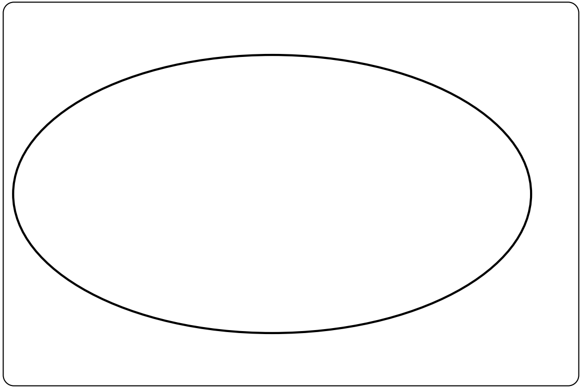
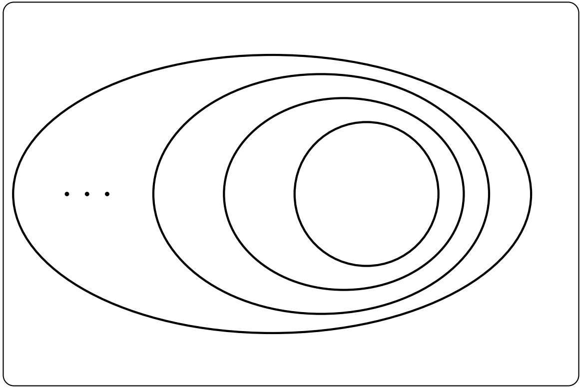
$$P(X|Y) > t.$$

- Trivially, the empty set of P-stable^t.
- If $P(X) = 1$, then X is P-stable^t.
- There are P-stable^t sets with $0 < P(X) < 1$.



- Assuming countable additivity and $t \geq \frac{1}{2}$, The class of P-stable^t propositions X in $\mathfrak{A}$ with $P(X) < 1$ is well-ordered with respect to the subset relation.
- If there is a non-empty P-stable^r $X \in \mathfrak{A}$ with $P(X) < 1$, then there is also a least such X.



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$w \in SB(H)$ iff for all $E \in \mathfrak{A}(W)$ with $H \cap E \neq \emptyset$ and $P(E) \neq 0$: $P(H \mid E) \geq t$

$w \in SB(H)$ iff for all $E \in \mathfrak{A}(W)$ with $H \cap E \neq \emptyset$ and $P(E) \neq 0$: $P(H \mid E) \geq t_C$

1. The threshold t is determined contextually
   (the "cautiousness level")

$w \in SB(H)$ iff for all $E \in \mathfrak{A}_H(W)$ with $H \cap E \neq \emptyset$ and $P(E) \neq 0$:
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1. The threshold t is determined contextually
   (the "cautiousness level")

2. The evidence "relevant" to H

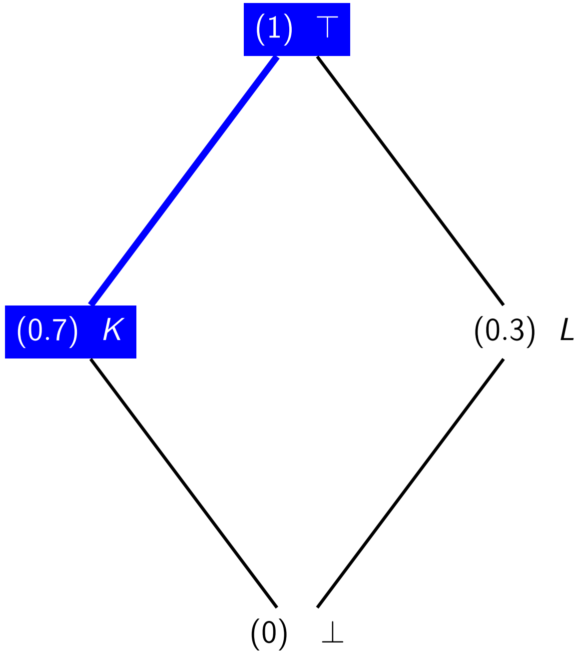
$w \in SB(H)$ iff for all $E \in \mathfrak{A}_H(W_\Pi)$ with $H \cap E \neq \emptyset$ and $P(E) \neq 0$:
$P(H \mid E) \geq t_C$

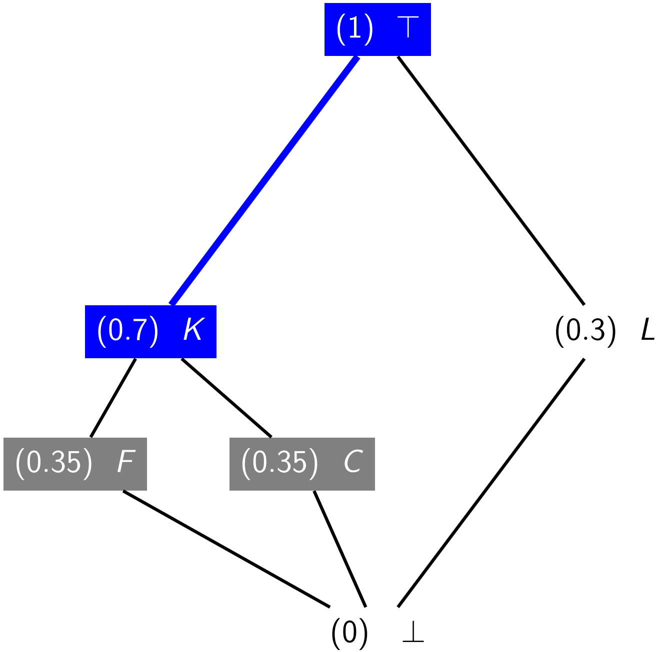
1. The threshold t is determined contextually (the "cautiousness level")
2. The evidence "relevant" to H
3. The states may be contextually determined (by a partition Π on a set W of "maximally specific worlds")

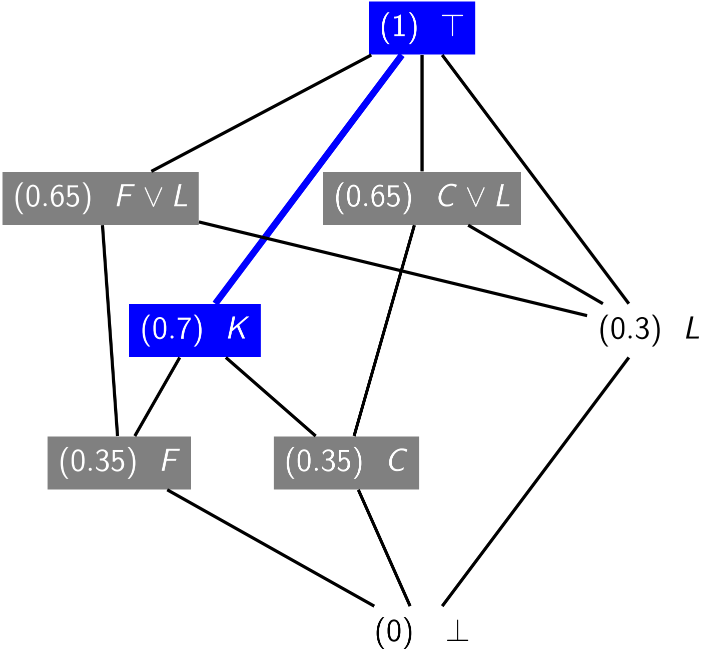
H. Leitgeb. *The Stability Theory of Belief*. The Philosophical Review 123/2, 131-171, 2014.

H. Leitgeb. *The Humean Thesis on Belief*. Proceedings of the Aristotelian Society of Philosophy 89(1), 143-185, 2015.

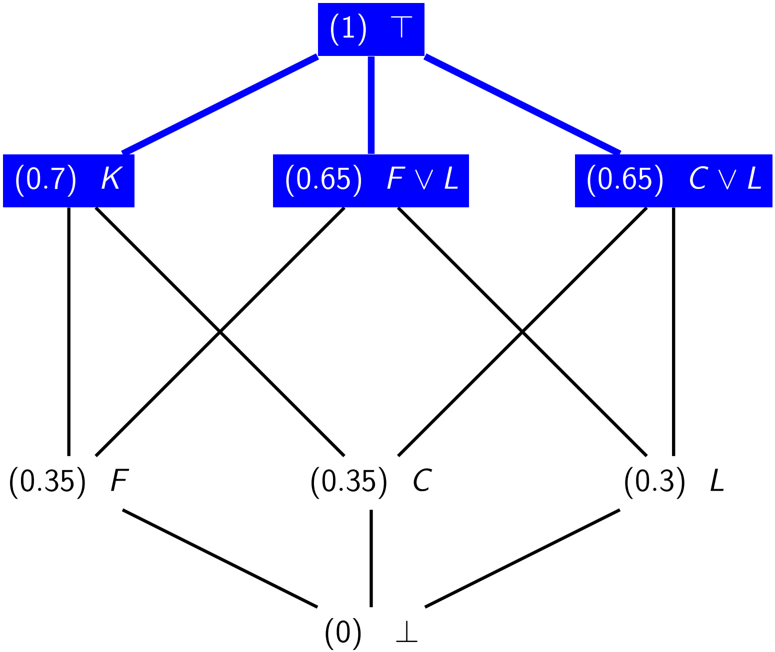
R. Pettigrew. *Pluralism about belief states*. Proceedings of the Aristotelian Society 89(1):187-204, 2015.

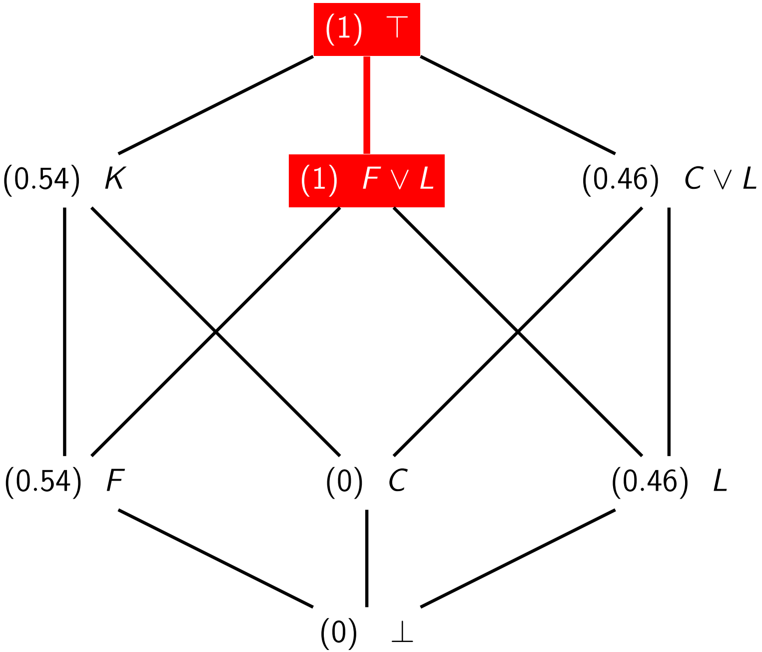

(1) ⊤
(0.7) K
(0.3) L
(0) ⊥


(1) ⊤
(0.7) K
(0.3) L
(0.35) F
(0.35) C
(0) ⊥



(1) ⊤
(0.65) $F \vee L$
(0.65) $C \vee L$
(0.7) K
(0.3) L
(0.35) F
(0.35) C
(0) ⊥


(1) $\top$
(0.7) K
(0.65) $F \vee L$
(0.65) $C \vee L$
(0.35) F
(0.35) C
(0.3) L
(0) $\bot$



(1) $\top$
(0.54) K
(1) $F \vee L$
(0.46) $C \vee L$
(0.54) F
(0) C
(0.46) L
(0) $\bot$

Thus, while **stable belief** is stable under acquisition of new (doxastically possible) evidence and Lockean belief is not, **stable belief** is not stable under fine-graining of possibilities while Lockean belief is.

Leitgeb's Solution to the Lottery Paradox

In a context in which the agent is interested in *whether ticket i will be drawn*; for example, for $i = 1$: Let Π be the corresponding partition:

$$\{\{w_1\}, \{w_2, \ldots, w_{1,000,000}\}\}$$

The resulting probability measure P_Π is given so that P is given by P so that:

$$P_\Pi(\{\{w_1\}\}) = \frac{1}{1,000,000} \qquad P_\Pi(\{\{w_2, \ldots, w_{1,000,000}\}\}) = \frac{999,999}{1,000,000}$$

There are two P_Π-stable sets, and one of the two possible choices for the strongest believed proposition $B_W^\Pi = \{\{w_2, \ldots, w_{1{,}000{,}000}\}\}$.

If B_W^Π is chosen as such, our perfectly rational agent believes of ticket $i = 1$ that it will not be drawn, (and of course P1 -P3 are satisfied).

For example, this might be a context in which a single ticket holder—the person holding ticket 1—would be inclined to say of his or her ticket: "I believe it won't win."

In a context in which the agent is interested in *which ticket will be drawn*: Let Π' be the corresponding partition that consists of all singleton subsets of W. The probability measure P^{Π} is the uniform probability on W.

The only P-stable set—and hence the only choice for the strongest believed proposition $B_W^{\Pi'}$—is W itself: our perfectly rational agent believes that some ticket will be drawn, but he or she does not believe of any ticket that it will not win

For example, this might be a context in which a salesperson of tickets in a lottery would be inclined to say of each ticket: "It might win" (that is, it is not the case that I believe that it won't win).

In either of the two contexts from before, the theory avoids the absurd conclusion of the Lottery Paradox; in each context, it preserves the closure of belief under conjunction; and in each context, it preserves the Lockean thesis for some threshold ($r = \frac{999{,}999}{1{,}000{,}000}$ in the first case, $r = 1$ in the second case)-all of this follows from P-stability and the theorem.

In the first Π-context, the intuition is preserved that, in some sense, one believes of ticket i that it will lose since it is so likely to lose.

In the second Π'-context, the intuition is preserved that, in a different sense, one should not believe of any ticket that it will lose since the situation is symmetric with respect to tickets, as expressed by the uniform probability measure, and of course some ticket must win.

Finally, by disregarding or mixing the contexts, it becomes apparent why one might have regarded all of the premises of the Lottery Paradox as true.

But according to the present theory, contexts should not be disregarded or mixed: partitions Π and Π' differ from each other, and different partitions may lead to different beliefs, as observed in the last section and as exemplified in the Lottery Paradox.

Conditioning

$$p_0 \xRightarrow{\text{Learn that } E} p_t(\cdot) = p_0(\cdot \mid E)$$

Conditional Probability

The probability of E *given* F, dented $p(E|F)$, is defined to be

$$p(E|F) = \frac{p(E \cap F)}{p(F)}.$$

provided $P(F) > 0$.

Setting $p_t(\cdot) = p_0(\cdot \mid E)$ is demonstrably the correct thing to do just in case, for all propositions $H \in \Sigma$, both:

1. Certainty: $p_t(E) = 1$
2. Rigidity: $p_t(H \mid E) = p_0(H \mid E)$

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Observation by candlelight

An agent inspects a piece of cloth by candlelight, and gets the impression that it is green (G), although he concedes that it might be blue (B) or even (but very improbably) violet (V).

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$$\Downarrow$$

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Is there a proposition E such that $p_t(\cdot) = p_0(\cdot \mid E)$?

Jeffrey Conditionalization

When an observation bears directly on the probabilities over a partition $\{E_i\}$, changing them from $p(E_i)$ to $q(E_i)$, the new probability for any proposition H should be

$$q(H) = \sum_i p(H \mid E_i) q(E_i)$$

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Fact: If q is obtained from p by Jeffrey Conditioning on the partition $\{E, \overline{E}\}$ with $q(E) = 1$, then $q(\cdot) = p(\cdot \mid E)$.