

Reasoning in Games

Lecture 2

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Plan

- ✓ Day 1: Decision Theory
- ▶ Day 2: Games and Game Models
- ▶ Day 3: Modeling Deliberation (in Games)
- ▶ Day 4: Backward and Forward Induction
- ▶ Day 5: Spill Over, Concluding Remarks (Language-Based Games/
Variable Frame Theory, Behavioral Game Theory, ...)

Taking Stock

- ▶ Many choice rules: MEU, strict/weak dominance, maxmin, minmax regret
 - Which one is “best”?
 - What are the relationships between the different choice rules?
- ▶ Payoff is not the same as utility (von Neumann-Morgenstern utilities)
- ▶ Rational choice models should be applied with care (act-state dependence, deliberation, attitudes towards risk, attitudes toward ambiguity, . . .)

From Decisions to Games, I

Commenting on the difference between Robin Crusoe's maximization problem and the maximization problem faced by participants in a social economy, von Neumann and Morgenstern write:

“Every participant can determine the variables which describe his own actions but not those of the others. Nevertheless those “alien” variables cannot, from his point of view, be described by statistical assumptions.

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“Every participant can determine the variables which describe his own actions but not those of the others. Nevertheless those “alien” variables cannot, from his point of view, be described by statistical assumptions. This is because the others are guided, just as he himself, by rational principles—whatever that may mean—and no *modus procedendi* can be correct which does not attempt to understand those principles and the interactions of the conflicting interests of all participants.”

(vNM, pg. 11)

Game Situations

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

	Bob	
<i>L</i>		<i>R</i>
	1	0

1. a group of *self-interested* agents (players) involved in some interdependent *decision problem*

Game Situations

	Bob	
<i>L</i>		<i>R</i>
	1	0
	0	1

1. a group of *self-interested* agents (players) involved in some interdependent *decision problem*

Game Situations

		Bob	
		L	R
Ann	U	1 1	0 0
	D	0 0	1 1

1. a **group** of *self-interested* agents (players) involved in some interdependent **decision problem**

Game Situations

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

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Game Situations

		Bob	
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1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Just Enough Game Theory

A **game** is a mathematical model of a strategic interaction that includes

- ▶ the actions the players *can* take
- ▶ the players' interests (i.e., preferences),
- ▶ the “structure” of the decision problem

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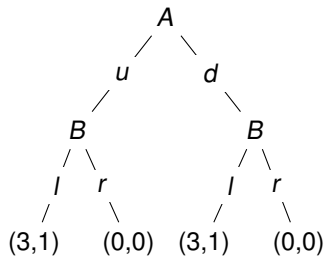
It does not specify the actions that the players do take.

Games

		<i>B</i>	
		l	r
<i>A</i>	u	3, 1	0, 0
	d	0, 0	1, 3

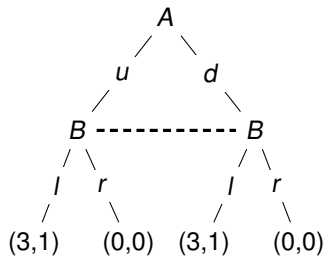
Games

		<i>B</i>	
		<i>l</i>	<i>r</i>
<i>A</i>	<i>u</i>	3, 1	0, 0
	<i>d</i>	0, 0	1, 3



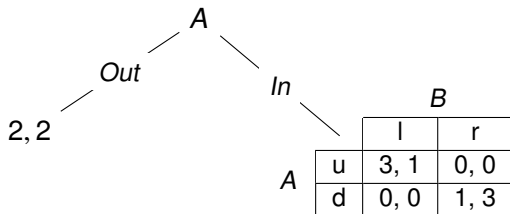
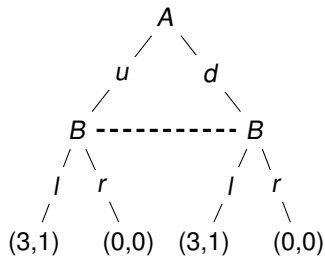
Games

		<i>B</i>	
		<i>l</i>	<i>r</i>
<i>A</i>	<i>u</i>	3, 1	0, 0
	<i>d</i>	0, 0	1, 3



Games

		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3



Questions

- ▶ Do players maximize (expected) utilities when playing games?

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 - How, exactly, do you apply [revealed preference theory](#) to game theory?
 - How, exactly, do you apply [von Neumann-Morgenstern utility theory](#) to game theory?
 - How, exactly, do you apply [Savage's subjective expected utility theory](#) to game theory?
 - How, exactly, do you apply [Kahneman and Tversky's prospect theory](#) to game theory?

Questions

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 - How, exactly, do you apply [Kahneman and Tversky's prospect theory](#) to game theory?

- ▶ What is game theory trying to accomplish?
(predictions? recommendations? explanations? analytical results?)

I. Gilboa and D. Schmeidler. *A Derivation of Expected Utility Maximization in the Context of a Game*. Games and Economic Behavior, 44, pgs. 184 - 194, 2003.

M. Mariotti. *Decisions in games: why there should be a special exemption from Bayesian rationality*. Journal of Economic Methodology, 4: 1, pgs. 43 - 60, 1997.

P. Hammond. *Expected Utility in Non-Cooperative Game Theory*. in *Handbook of Utility Theory*, 2004.

J. Kadane and P. Larkey. *Subjective Probability and the Theory of Games*. Management Science, Volume 28, 1982.

From Decisions to Games, II

“[T]he fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play.”

R. Aumann and J. Dreze. *Rational Expectations in Games*. *American Economic Review*, 98, pp. 72-86, 2008.

The Guessing Game



Guess a number between 1 & 100.
The closest to $\frac{2}{3}$ of the average wins.

The Guessing Game



Guess a number between 1 & 100.
The closest to $\frac{2}{3}$ of the average wins.

What number should you guess?

The Guessing Game



Guess a number between 1 & 100.
The closest to $\frac{2}{3}$ of the average wins.

What number should you guess? 100

The Guessing Game



Guess a number between 1 & 100.
The closest to $2/3$ of the average wins.

What number should you guess? ~~100~~, 99

The Guessing Game



Guess a number between 1 & 100.
The closest to $2/3$ of the average wins.

What number should you guess? ~~100~~, ~~99~~, ..., 67

The Guessing Game



Guess a number between 1 & 100.
The closest to $2/3$ of the average wins.

What number should you guess? ~~100~~, ~~99~~, ..., ~~67~~, ..., 2, 1

The Guessing Game



Guess a number between 1 & 100.
The closest to $2/3$ of the average wins.

What number should you guess? ~~100~~, ~~99~~, ..., ~~67~~, ..., ~~2~~, **1**

Solution Concepts

A **solution concept** is a systematic description of the outcomes that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards induction, or iterated dominance of various kinds.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Let $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ be a finite strategic game (each S_i is finite and the set of players N is finite).

A **strategy profile** is an element $\sigma \in S = S_1 \times \cdots \times S_n$

σ is a **Nash equilibrium** provided for all i , for all $s_i \in S_i$,

$$u_i(\sigma) \geq u_i(s_i, \sigma_{-i})$$

Zero-Sum Games

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2

What should Ann *do*?

Zero-Sum Games

		Bob	
		L	R
Ann	U	1,4	4,1
	D	2,3	3,2

What should Ann *do*? *Bob best choice in Ann's worst choice*

Zero-Sum Games

		Bob		
		L	R	
Ann	U	1,4	4,1	1
	D	2,3	3,2	2

What should Ann do? *Security strategy: minimize over each row and choose the maximum value*

Zero-Sum Games

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2
		3	1

What should Bob do? *Security strategy: minimize over each column and choose the maximum value*

Zero-Sum Games

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2

The profile of security strategies (D, L) is a Nash equilibrium

Matching Pennies

		Bob	
		<i>H</i>	<i>T</i>
Ann	<i>H</i>	1, -1	-1, 1
	<i>T</i>	-1, 1	1, -1

There are no pure strategy Nash equilibria.

Mixed Strategies

		Bob	
		<i>H</i>	<i>T</i>
Ann	<i>H</i>	1, -1	-1, 1
	<i>T</i>	-1, 1	1, -1

A **mixed strategy** is a probability distribution over the set of pure strategies. For instance:

- ▶ $(1/2H, 1/2T)$
- ▶ $(1/3H, 2/3T)$
- ▶ ...

Matching Pennies

		Bob	
		<i>H</i>	<i>T</i>
Ann	<i>H</i>	1, -1	-1, 1
	<i>T</i>	-1, 1	1, -1

The mixed strategy $([1/2 : H, 1/2 : T], [1/2 : H, 1/2 : T])$ is the only Nash equilibrium.

Theorem (Von Neumann). For every two-player zero-sum game with finite strategy sets S_1 and S_2 , there is a number v , called the **value** of the game such that:

1. $v = \max_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q) = \min_{q \in \Delta(S_2)} \max_{p \in \Delta(S_1)} U_1(p, q)$
2. The set of mixed Nash equilibria is nonempty. A mixed strategy profile (p, q) is a Nash equilibrium if and only if

$$p \in \operatorname{argmax}_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q)$$

$$q \in \operatorname{argmax}_{q \in \Delta(S_2)} \min_{p \in \Delta(S_1)} U_1(p, q)$$

3. For all mixed Nash equilibria (p, q) , $U_1(p, q) = v$

Why play such an equilibrium?

“Let us now imagine that there exists a complete theory of the zero-sum two-person game which tells a player what to do, and which is absolutely convincing. If the players knew such a theory then each player would have to assume that his strategy has been “found out” by his opponent. The opponent knows the theory, and he knows that the player would be unwise not to follow it... a satisfactory theory can exist only if we are able to harmonize the two extremes...strategies of player 1 ‘found out’ or of player 2 ‘found out.’ ” (pg. 148)

J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1944.

“Von Neumann and Morgenstern are assuming that the *payoff matrix* is common knowledge to the players, but presumably the players’ subjective probabilities might be private. Then each player might quite reasonably act to maximize subjective expected utility, believing that he will *not* be found out, with the result *not* being a Nash equilibrium.”

(Skyrms, pg. 14)

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2

- ▶ Suppose that Ann believes Bob will play *L* with probability $1/4$, for *whatever reason*. Then,

$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,4	4,1
	<i>D</i>	2,3	3,2

- ▶ Suppose that Ann believes Bob will play *L* with probability $1/4$, for whatever reason. Then,

$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$

- ▶ But, *L* maximizes expected utility no matter what belief Bob may have:

$$p + 3 = 4 \times p + 3 \times (1 - p) \geq 1 \times p + 2 \times (1 - p) = 2 - p$$

In zero-sum games

- ▶ There exists a mixed strategy Nash equilibrium
- ▶ There may be more than one Nash equilibria
- ▶ Security strategies are always a Nash equilibrium
- ▶ Components of Nash equilibria are interchangeable: If σ and σ' are Nash equilibria in a 2-player game, then (σ_1, σ'_2) is also a Nash equilibria.

Finding the rational choice...

		Bob	
		<i>H</i>	<i>T</i>
Ann	<i>H</i>	1,-1	-1,1
	<i>T</i>	-1,1	1,-1

What is a rational choice for Ann (Bob)?

Finding the rational choice...

		Bob	
		<i>H</i>	<i>T</i>
Ann	<i>H</i>	1,-1	-1,1
	<i>T</i>	-1,1	1,-1

What is a rational choice for Ann (Bob)? *Flip a coin!*

Finding the rational choice...

		Bob	
		C1	C2
Ann	P1	1,-1	-1,1
	P2	-1,1	1,-1

What is a rational choice for Ann (Bob)?

Finding the rational choice...

		Bob	
		C1	C2
Ann	P1	1,-1	-1,1
	P2	-1,1	1,-1

		Bob	
		C1	C2
Ann	P1	1,-1	1,-1
	P2	1,-1	1,-1

What is a rational choice for Ann (Bob)? *Play a different game!*

Let $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ be a finite strategic game.

$$\Sigma_i = \{p \mid p : S_i \rightarrow [0, 1] \text{ and } \sum_{s_i \in S_i} p(s_i) = 1\}$$

The **mixed extension** of G is the game $\langle \{\Sigma_i\}_{i \in N}, \{U_i\}_{i \in N} \rangle$ where for $\sigma \in \Sigma = \Sigma_1 \times \cdots \times \Sigma_n$:

$$U_i(\sigma) = \sum_{(s_1, \dots, s_n) \in S} \sigma_1(s_1) \sigma_2(s_2) \cdots \sigma_n(s_n) u_i(s_1, \dots, s_n)$$

Theorem. Suppose that σ is a Nash equilibrium in mixed strategies for a game $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$. Suppose that $s_i, s_i^* \in S_i$ are two pure strategies such that $\sigma_i(s_i) > 0$ and $\sigma_i(s_i^*) > 0$, then

$$U_i(s_i, \sigma_{-i}) = U_i(s_i^*, \sigma_{-i})$$

Theorem (Nash). Every finite game G has a Nash equilibrium in mixed strategies (i.e., there is a Nash equilibrium in the mixed extension G).

Not all equilibrium are created equal...

Perfect equilibrium

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	0,0

Perfect equilibrium

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	0,0

Perfect equilibrium

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	0,0

Isn't (U, L) more "reasonable" than (D, R) ?

Perfect equilibrium

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

Perfect equilibrium

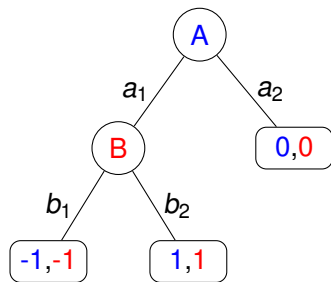
		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

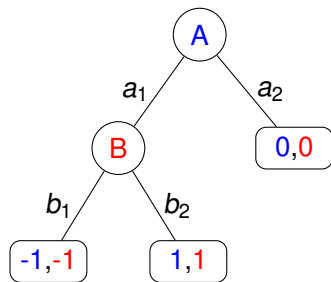
ϵ -perfect equilibrium: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than ϵ

Perfect equilibrium: the mixed strategy profile that is the limit as ϵ goes to 0 of ϵ -perfect equilibria.

Normal form vs. Extensive form



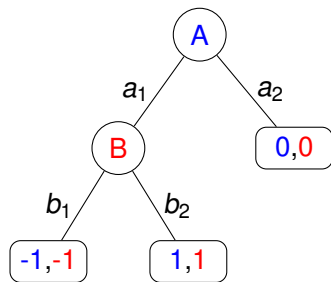
Normal form vs. Extensive form



b_1 if a_1 b_2 if a_1

a_1	$-1,-1$	$1,1$
a_2	$0,0$	$0,0$

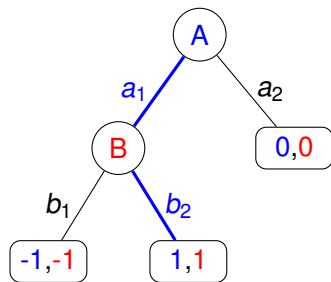
Normal form vs. Extensive form



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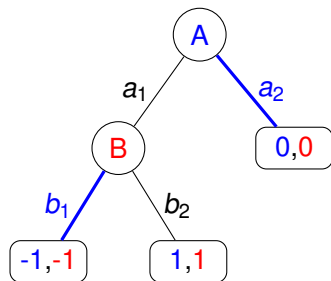
Normal form vs. Extensive form



b_1 if a_1 b_2 if a_1

a_1	$-1,-1$	$1,1$
a_2	$0,0$	$0,0$

Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
a_1	$-1, -1$	$1, 1$
a_2	$0, 0$	$0, 0$

(Cf. the various notions of *sequential equilibrium*)

Pure Coordination Game

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1,1	0,0
	<i>D</i>	0,0	1,1

The profiles **(U, L)** and **(D, R)** are Nash equilibria.

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do?

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning* is not **Pareto!**

Traveler's Dilemma

	2	3	4	...	99	100
2	(2, 2)	(4, 0)	(4, 0)	...	(4, 0)	(4, 0)
3	(0, 4)	(3, 3)	(5, 1)	...	(5, 1)	(5, 1)
4	(0, 4)	(1, 5)	(4, 4)	...	(6, 2)	(6, 2)
⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	(0, 4)	(1, 5)	(2, 6)	...	(99, 99)	(101, 97)
100	(0, 4)	(1, 5)	(2, 6)	...	(97, 101)	(100, 100)

Traveler's Dilemma

	2	3	4	...	99	100
2	(2, 2)	(4, 0)	(4, 0)	...	(4, 0)	(4, 0)
3	(0, 4)	(3, 3)	(5, 1)	...	(5, 1)	(5, 1)
4	(0, 4)	(1, 5)	(4, 4)	...	(6, 2)	(6, 2)
⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	(0, 4)	(1, 5)	(2, 6)	...	(99, 99)	(101, 97)
100	(0, 4)	(1, 5)	(2, 6)	...	(97, 101)	(100, 100)

(2, 2) is the only Nash equilibrium.

Traveler's Dilemma

	2	3	4	...	99	100
2	(2, 2)	(4, 0)	(4, 0)	...	(4, 0)	(4, 0)
3	(0, 4)	(3, 3)	(5, 1)	...	(5, 1)	(5, 1)
4	(0, 4)	(1, 5)	(4, 4)	...	(6, 2)	(6, 2)
⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	(0, 4)	(1, 5)	(2, 6)	...	(99, 99)	(101, 97)
100	(0, 4)	(1, 5)	(2, 6)	...	(97, 101)	(100, 100)

The analysis is insensitive to the amount of reward/punishment.

In an arbitrary (finite) strategic games

- ▶ There exists a mixed strategy Nash equilibrium
- ▶ Security strategies are not necessarily a Nash equilibrium
- ▶ There may be more than one Nash equilibrium
- ▶ Components of Nash equilibrium are not interchangeable.

“...no, equilibrium is not the way to look at games. Now, Nash equilibrium is king in game theory. Absolutely king. We say: No, Nash equilibrium is an interesting concept, and it's an important concept, but it's not the most basic concept. The most basic concept should be: to maximise your utility given your information. It's in a game just like in any other situation. Maximise your utility given your information!”

Robert Aumann, 5 Questions on Epistemic Logic, 2010

Rationalizability

“Analysis of strategic economic situations requires us, implicitly or explicitly, to maintain as plausible certain psychological hypotheses. They hypothesis that real economic agents universally recognize the salience of Nash equilibria may well be less accurate than, for example, the hypothesis that agents attempt to “out-smart” or “second-guess” each other, believing that their opponents do likewise.” (pg. 1010)

B. D. Bernheim. *Rationalizable Strategic Behavior*. *Econometrica*, 52:4, pgs. 1007 - 1028, 1984.

Rationalizability

“The rules of a game and its numerical data are seldom sufficient for logical deduction alone to single out a unique choice of strategy for each player. *To do so one requires either richer information (such as institutional detail or perhaps historical precedent for a certain type of behavior) or bolder assumptions about how players choose strategies.* Putting further restrictions on strategic choice is a complex and treacherous task. But one’s intuition frequently points to patterns of behavior that cannot be isolated on the grounds of consistency alone.”
(pg. 1035)

D. G. Pearce. *Rationalizable Strategic Behavior*. *Econometrica*, 52, 4, pgs. 1029 - 1050, 1984.

		Bob		
		<i>L</i>	<i>C</i>	<i>R</i>
Ann	<i>T</i>	1, -1	0,0	-1, 1
	<i>M</i>	0,0	0,0	0,0
	<i>B</i>	-1,1	0,0	1,-1

(M, C) is the unique Nash equilibria. Suppose that both player's subjective probabilities are $(1/3, 1/3, 1/3)$, and this is *common knowledge*. Then, any choice maximizes the players' expected utility.

Suppose that $G = (S_1, \dots, S_n, u_1, \dots, u_n)$ is a strategic game.

A strategy $s_i \in S_i$ is a **best response** to a joint probability $m_{-i} \in \prod_{j \neq i} \Delta(S_j)$ iff $U_i(s_i, m_{-i}) \geq U_i(s'_i, m_{-i})$ for all $s'_i \in S_i$ (here $U_i(\cdot, m_{-i})$ is the expected utility with respect to the joint probability m_{-i}).

Reasoning in Games

What is *strategic* reasoning?

What are the players reasoning *about*? What they should do? What their opponents are going to do? What their opponents are thinking? Their preferences? The model?

Knowledge and beliefs in game situations

J. Harsanyi. *Games with incomplete information played by "Bayesian" players I-III. Management Science Theory* **14**: 159-182, 1967-68.

R. Aumann. *Interactive Epistemology I & II. International Journal of Game Theory* (1999).

P. Battigalli and G. Bonanno. *Recent results on belief, knowledge and the epistemic foundations of game theory. Research in Economics* (1999).

R. Myerson. *Harsanyi's Games with Incomplete Information. Special 50th anniversary issue of Management Science*, 2004.

John C. Harsanyi, nobel prize winner in economics, developed a theory of games with **incomplete information**.

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1. **incomplete information**: uncertainty about the *structure* of the game (outcomes, payoffs, strategy space)
2. **imperfect information**: uncertainty *within the game* about the previous moves of the players

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Epistemic Game Theory

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The difference lies in the attitudes of the players, in their expectations about each other, in custom, and in history, though the rules of the game do not distinguish between the two situations. (pg. 72)

R. Aumann and J. H. Dreze. *Rational Expectations in Games*. American Economic Review 98 (2008), pp. 72-86.

The Epistemic Program in Game Theory

“...the analysis constitutes a fleshing-out of the textbook interpretation of equilibrium as ‘rationality plus correct beliefs.’...this suggests that equilibrium behavior cannot arise out of strategic reasoning alone. ”

E. Dekel and M. Siniscalchi. *Epistemic Game Theory*. manuscript, 2013.

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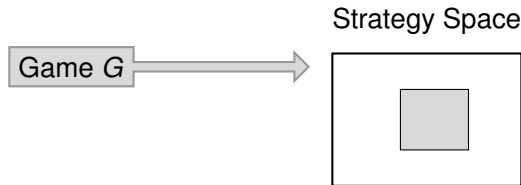
EP and O. Roy. *Epistemic Game Theory*. Stanford Encyclopedia of Philosophy, 2015.

The Epistemic Program in Game Theory

Game G

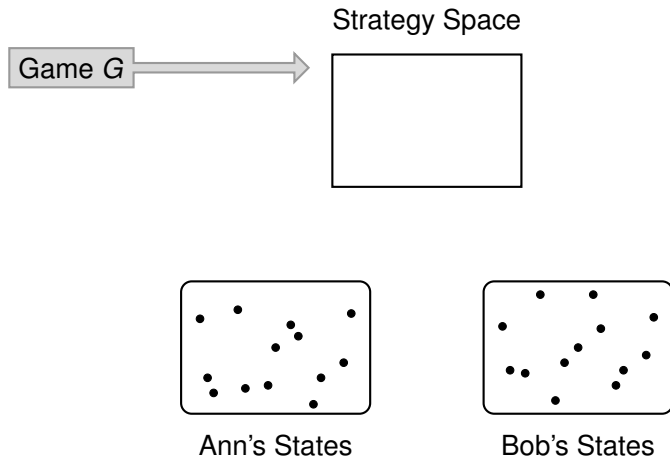
G : available actions, payoffs, structure of the decision problem

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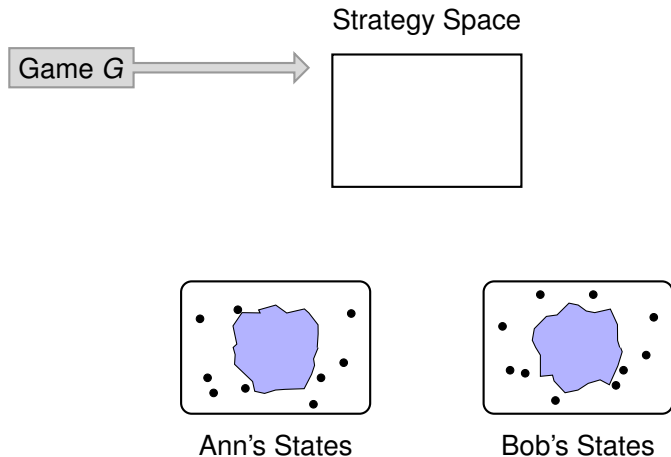
solution concepts are systematic descriptions of what players *do*

The Epistemic Program in Game Theory



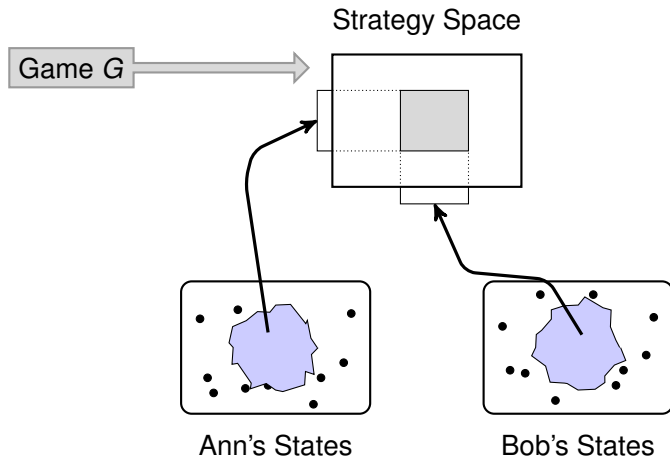
The game model includes *information states* of the players

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Restrict to information states satisfying some rationality condition

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Project onto the strategy space

Tomorrow: Game Models and Deliberation