Probabilistic Methods in Social Choice

Eric Pacuit
University of Maryland

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Plan

⇒ Background on voting theory
- Generating preference profiles
- Quantitative analysis of voting methods
- Probabilistic voting methods
- Condorcet jury theorem and related results
- Aggregating probabilistic judgements

Positional scoring rules

A *scoring vector* is a vector $\langle s_1, \ldots, s_n \rangle$ of numbers such that for each $m \in \{1, \ldots, n-1\}$, $s_m \geq s_{m+1}$.

Given a profile $\mathbf{P}$ with $|X(\mathbf{P})| = n$, $x \in X(\mathbf{P})$, a scoring vector $\vec{s}$ of length n, and $i \in V(\mathbf{P})$, define $score_{\vec{s}}(x, \mathbf{P}_i) = s_r$ where $r = Rank(x, \mathbf{P}_i)$.

Let $score_{\vec{s}}(x, \mathbf{P}) = \sum_{i \in V(\mathbf{P})} score_{\vec{s}}(x, \mathbf{P}_i)$. A voting method F is a **positional scoring rule** if there is a map $\mathcal{S}$ assigning to each natural number n a scoring vector of length n such that for any profile $\mathbf{P}$ with $|X(\mathbf{P})| = n$,

$$F(\mathbf{P}) = \mathrm{argmax}_{x \in X(\mathbf{P})} score_{\mathcal{S}(n)}(x, \mathbf{P}).$$

Examples

Borda:	$\mathcal{S}(n) = \langle n-1, n-2, \ldots, 1, 0 \rangle$
Plurality:	$\mathcal{S}(n) = \langle 1, 0, \ldots, 0 \rangle$
Anti-Plurality:	$\mathcal{S}(n) = \langle 1, 1, \ldots, 1, 0 \rangle$

1	3	2	4
a	*b*	*b*	*c*
c	*a*	*c*	*a*
b	*c*	*a*	*b*

Borda winner	*c*
Plurality winner	*b*
Anti-Plurality winner	*a*

Iterative procedures: Instant Runoff

- If some alternative is ranked first by an absolute majority of voters, then it is declared the winner.
- Otherwise, the alternative ranked first be the fewest voters (the plurality loser) is eliminated.
- Votes for eliminated alternatives get transferred: delete the removed alternatives from the ballots and "shift" the rankings (e.g., if 1st place alternative is removed, then your 2nd place alternative becomes 1st).

Also known as Ranked-Choice, STV, Hare

How should you deal with ties? (e.g., multiple alternatives are plurality losers)

Iterative procedures

Variants:

- Plurality with runoff: remove all candidates except top two plurality score;
- Coombs: remove candidates with most last place votes;
- Baldwin: remove candidate with smallest Borda score;
- Nanson: remove candidates with below average Borda score

Example

1	1	1	1	1
c	b	a	b	d
a	d	b	c	a
d	a	c	d	b
b	c	d	a	c

Instant Runoff	$\{b\}$
Plurality with Runoff	$\{a, b\}$
Coombs	$\{d\}$
Baldwin	$\{a, b, d\}$
Strict Nanson	$\{a\}$

Instant Runoff violates Majority Defeat

This example comes from ElectionScience.org as "a simplified approximation of what happened in the 2009 IRV mayoral election in Burlington, Vermont."

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Consider three elections for a Democrat d, Progressive p, and Republican r:

37	29	34
d	d	p
p	p	d

37	29	34
r	d	p
d	p	d
p	r	r

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On the left, the IRV winner is d.

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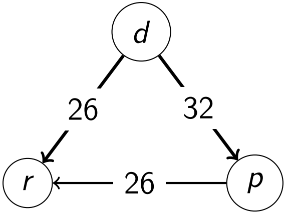
In the wake of this election, IRV was repealed in Burlington. . . .

37	29	34
r	d	p
d	p	d
p	r	r



Condorcet criteria

The **Condorcet winner** in a profile $\mathbf{P}$ is a candidate $x \in X(\mathbf{P})$ that is the maximum of the majority ordering, i.e., for all $y \in X(\mathbf{P})$, if $x \neq y$, then $Margin_{\mathbf{P}}(x, y) > 0$.

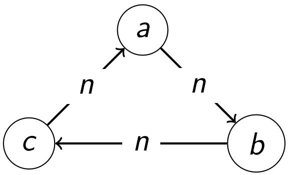
The **Condorcet loser** in a profile $\mathbf{P}$ is a candidate $x \in X(\mathbf{P})$ that is the minimum of the majority ordering, i.e., for all $y \in X(\mathbf{P})$, if $x \neq y$, then $Margin_{\mathbf{P}}(y, x) < 0$.

A voting method F is **Condorcet consistent**, if for all $\mathbf{P}$, if x is a Condorcet winner in $\mathbf{P}$, then $F(\mathbf{P}) = \{x\}$.

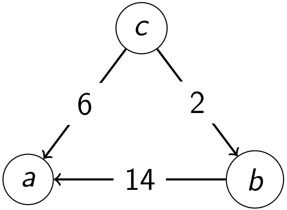
A voting method F is susceptible to the **Condorcet loser paradox** (also known as *Borda's paradox*) if there is some $\mathbf{P}$ such that x is a Condorcet loser in $\mathbf{P}$ and $x \in F(\mathbf{P})$.

Condorcet paradox

n	n	n
a	b	c
b	c	a
c	a	b



20	13	21	14	22	10
a	*a*	*c*	*b*	*b*	*c*
c	*b*	*b*	*a*	*c*	*a*
b	*c*	*a*	*c*	*a*	*b*



Condorcet winner: *c*

Instant Runoff winner: *b*

Plurality with Runoff winner: *b*

Plurality winner: *b*

Borda winner: *b*

Theorem (Smith 1973, Young 1974)

A voting method satisfies Anonymity, Neutrality and **Reinforcement** *if and only if F is a scoring rule.*

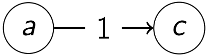
Saari's argument, Balinski and Laraki (2010, pg. 77); Zwicker (2016, Proposition 2.5): Multiple districts paradox, f *cancels properly*.

2	2	2	1	2
a	b	c	a	b
b	c	a	b	a
c	a	b	c	c

- no Condorcet winner in the left profile
- b is the Condorcet winner in the right profile
- a is the Condorcet winner in the combined profiles

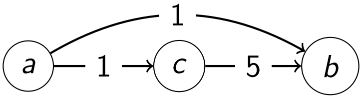
Spoilers

2	3
c	a
a	c



Borda winner: a

2	3
c	a
b	c
a	b



Borda winner: c

A *stability* property of Condorcet winners:

- if a candidate a would be the Condorcet winner without another candidate b in the election, and a beats b in a head-to-head majority comparison, then a is still the Condorcet winner in the election with b included.

Condorcet consistent voting methods

- Minimax
- Copeland
- Beat Path
- Ranked Pairs
- Split Cycle

Minimax: For a profile $\boldsymbol{P}$, The Minimax winners in $\boldsymbol{P}$ are:

$$\text{argmin}_{x \in X(\boldsymbol{P})} \text{max}\{Margin_{\boldsymbol{P}}(y,x) \mid y \in X(\boldsymbol{P})\}$$

Copeland/Llull: For $\alpha \in [0,1]$, the Copeland$_{\alpha}$ score of a in $\boldsymbol{P}$ is the number of $b \in X(\boldsymbol{P})$ such that $Margin_{\boldsymbol{P}}(a,b) > 0$ plus α times the number of $b \in X(\boldsymbol{P})$ such that $Margin_{\boldsymbol{P}}(a,b) = 0$. $Copeland(\boldsymbol{P})$ (resp. $Llull(\boldsymbol{P})$) is the set of candidates with maximal Copeland$_{1/2}$ (resp. $Copeland_1$) score in $\boldsymbol{P}$.

Schulze Beat Path

For $a, b \in X(\boldsymbol{P})$, a *path from a to b in* $\boldsymbol{P}$ is a sequence $\rho = x_1, \ldots, x_n$ of distinct candidates in $X(\boldsymbol{P})$ with $x_1 = a$ and $x_n = b$ such that for $1 \leq k \leq n-1$, $Margin_{\boldsymbol{P}}(x_k, x_{k+1}) > 0$.

The *strength of* ρ is $\min\{Margin_{\boldsymbol{P}}(x_k, x_{k+1}) \mid 1 \leq k \leq n-1\}$.

Then a defeats b in $\boldsymbol{P}$ according to Beat Path if the strength of the strongest path from a to b is greater than the strength of the strongest path from b to a.

$BP(\boldsymbol{P})$ is the set of undefeated candidates.

Tideman Ranked Pairs, I

For a profile $\boldsymbol{P}$ and $T \in \mathcal{L}\big(\{(x, y) \mid x \neq y \text{ and } Margin_{\boldsymbol{P}}(x, y) \geq 0\}\big)$, called the *tie-breaking ordering*

A pair (x, y) of candidates has a *higher priority* than a pair (x', y') of candidates according to T when either $Margin_{\boldsymbol{P}}(x, y) > Margin_{\boldsymbol{P}}(x', y')$ or $Margin_{\boldsymbol{P}}(x, y) = Margin_{\boldsymbol{P}}(x', y')$ and $(x, y)\ T\ (x', y')$.

Tideman Ranked Pairs, II

We construct a *Ranked Pairs ranking* $\succ_{\boldsymbol{P},T} \in \mathcal{L}(X)$ as follows:

1. Initialize $\succ_{\boldsymbol{P},T}$ to $\varnothing$.
2. If all pairs (x, y) with $x \neq y$ and $Margin_{\boldsymbol{P}}(x, y) \geq 0$ have been considered, then return $\succ_{\boldsymbol{P},T}$. Otherwise let (a, b) be the pair with the highest priority among those with $a \neq b$ and $Margin_{\boldsymbol{P}}(a, b) \geq 0$ that have not been considered so far.
3. If $\succ_{\boldsymbol{P},T} \cup \{(a, b)\}$ is acyclic, then add (a, b) to $\succ_{\boldsymbol{P},T}$; otherwise, add (b, a) to $\succ_{\boldsymbol{P},T}$. Go to step 2.

When the procedure terminates, $\succ_{\boldsymbol{P},T}$ is a linear order.

The set $RP(\boldsymbol{P})$ of Ranked Pairs winners is the set of all $x \in X(\boldsymbol{P})$ such that x is the maximum of $\succ_{\boldsymbol{P},T}$ for some tie-breaking ordering T.

Tideman Ranked Pairs, III

Since calculating $RP(\boldsymbol{P})$ is an NP-complete problem, we also consider the non-anonymous version of Ranked Pairs proposed by Zavist and Tideman: use a distinguish voter's ranking to derive the tie-breaking ordering T.

Given $i \in V(\boldsymbol{P})$, let $T(\boldsymbol{P}_i)$ be the lexicographic order on $\{(x, y) \mid x \neq y \text{ and } Margin_{\boldsymbol{P}}(x, y) \geq 0\}$ derived from $\boldsymbol{P}_i$.

Since different profiles have different sets of voters, we cannot use the same distinguished voter for all profiles. Given a linear order L of $\mathcal{V}$ (the set of all possible voters), for any profile $\boldsymbol{P}$, we define $RPZT_L(\boldsymbol{P})$ to be the set of all $x \in X(\boldsymbol{P})$ such that x is the maximum of $\succ_{\boldsymbol{P}, T(\boldsymbol{P}_i)}$ where i is the minimal element of $V(\boldsymbol{P})$ according to L.

Split Cycle

Split Cycle defeat: a candidate *a* defeats a candidate *b* just in case

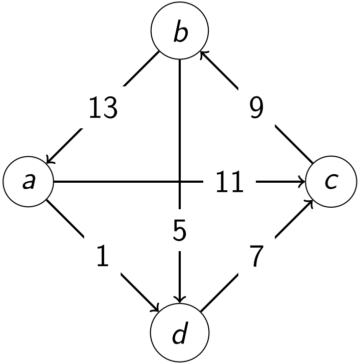
- the majority margin of *a* over *b* is greater than 0, and
- for every majority cycle containing *a* and *b*, the margin of *a* over *b* is greater than the smallest margin between consecutive candidates in the cycle.

The Split Cycle winners are the undefeated candidates.

An intuitive way defeat relation is as follows:

1. In each majority cycle, identify the wins with the smallest margin in that cycle.
2. After completing step 1 for all cycles, discard the identified wins. All remaining wins count as defeats.

Example



Minimax:	$\{d\}$
Copeland:	$\{a, b\}$
Beat Path:	$\{d\}$
Ranked Pairs:	$\{b\}$
Split Cycle:	$\{b, d\}$

We are interested in voting methods that:

1. respond in a reasonable way to **new candidates** joining the election;
2. respond in a reasonable way to **new voters** joining the election.

Background: Choice Consistency

Suppose that C is a choice function on X: for all $\emptyset \neq A \subseteq X$, $\emptyset \neq C(A) \subseteq A$.

Sen's α condition: if $A' \subseteq A$, then $C(A) \cap A' \subseteq C(A')$

Sen's γ condition (expansion): $C(A) \cap C(A') \subseteq C(A \cup A')$

Theorem (Sen 1971)

Let C be a choice function on a nonempty finite set X. TFAE:

1. *C satisfies α and γ*
2. *There exists a binary relation P on X such that for all $A \subseteq X$,*

$$C(A) = \{x \in A \mid \textit{there is no } y \in A \textit{ such that } y\, P\, x\}$$

A. Sen. *Choice Functions and Revealed Preference*. The Review of Economic Studies, 38:3, pp. 307-317, 1971.

Expansion in Voting

A **voting method** is a function F on the domain of all profiles such that for any profile $\mathbf{P}$, $\varnothing \neq F(\mathbf{P}) \subseteq X(\mathbf{P})$.

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A voting method F satisfies Expansion if for all profiles $\mathbf{P}$ and Y, Y' with $Y \cup Y' = X(\mathbf{P})$,

$$F(\mathbf{P}_{|Y}) \cap F(\mathbf{P}_{|Y'}) \subseteq F(\mathbf{P}).$$

Expansion in Voting

> First, it seems "intuitively right" that if x is a "winner" in both A and A', then it should stay a winner in $A \cup A'$. Second, it limits the manipulability of the SCF in that it implies that if x is a winner in A, and if B is formed by adding to A new alternatives (no matter whether they are winning or losing) such that x is a winner in some subset of B that contains the new alternatives, then x is still a winner in B. In particular, this means that one cannot turn x into a loser by introducing new alternatives to which x does not lose in duels. (p. 125)

G. Bordes. *On the Possibility of Reasonable Consistent Majoritarian Choice: Some Positive Results*. Journal of Economic Theory, 31:1, pp. 122 - 132, 1983.

Binary Expansion

Expansion: For all $A, A' \subseteq X$, $C(A) \cap C(A') \subseteq C(A \cup A')$.

Binary Expansion: For all $A, A' \subseteq X$ such that $|A'| = 2$, $C(A) \cap C(A') \subseteq C(A \cup A')$.

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Modulo α, Expansion is equivalent to Binary Expansion. Thus, we can replace Expansion by Binary Expansion in Sen's representation theorem.

Binary Expansion for Voting Methods

Expansion: For all profiles $\boldsymbol{P}$ and Y, Y' with $Y \cup Y' = X(\boldsymbol{P})$, $F(\boldsymbol{P}_{|Y}) \cap F(\boldsymbol{P}_{|Y'}) \subseteq F(\boldsymbol{P})$.

Binary Expansion: For all profiles $\boldsymbol{P}$ and Y, Y' with $Y \cup Y' = X(\boldsymbol{P})$, if $|Y'| = 2$, then $F(\boldsymbol{P}_{|Y}) \cap F(\boldsymbol{P}_{|Y'}) \subseteq F(\boldsymbol{P})$.

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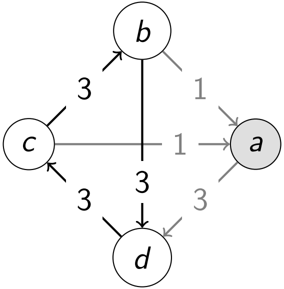
Strong Stability for Winners: For all profiles $\boldsymbol{P}$ and $a, b \in X(\boldsymbol{P})$,
if $a \in F(\boldsymbol{P}_{-b})$ and $Margin_{\boldsymbol{P}}(a, b) \geq 0$, then $a \in F(\boldsymbol{P})$.

W. Holliday and EP. *Split Cycle: A New Condorcet Consistent Voting Method Independent of Clones and Immune to Spoilers*. https://arxiv.org/abs/2004.02350, 2021.

Beat Path and Minimax Violate Binary Expansion

2	1	1	1	3	1	1	1
b	a	d	a	c	b	c	d
a	b	a	d	b	d	d	c
d	d	c	c	a	c	a	a
c	c	b	b	d	a	b	b



Beat Path and Minimax both violate Binary Expansion: $F(\boldsymbol{P}_{-a}) = \{b, c, d\}$, $Margin_{\boldsymbol{P}}(b, a) > 0$, and $b \notin F(\boldsymbol{P})$.

Variable Candidate Axioms

Binary Expansion/Strong Stability for Winners: For all profiles $\boldsymbol{P}$ and $a, b \in X(\boldsymbol{P})$, if $a \in F(\boldsymbol{P}_{-b})$ and $Margin_{\boldsymbol{P}}(a, b) \geq 0$, then $a \in F(\boldsymbol{P})$.

- **Stability for Winners**: For all profiles $\boldsymbol{P}$ and $a, b \in X(\boldsymbol{P})$, if $a \in F(\boldsymbol{P}_{-b})$ and $Margin_{\boldsymbol{P}}(a, b) > 0$, then $a \in F(\boldsymbol{P})$.
- **Immunity of Spoilers**: For all profiles $\boldsymbol{P}$ and $a, b \in X(\boldsymbol{P})$, if $a \in F(\boldsymbol{P}_{-b})$ and $Margin_{\boldsymbol{P}}(a, b) > 0$ and $b \notin F(\boldsymbol{P})$, then $a \in F(\boldsymbol{P})$.

We are interested in voting methods that:

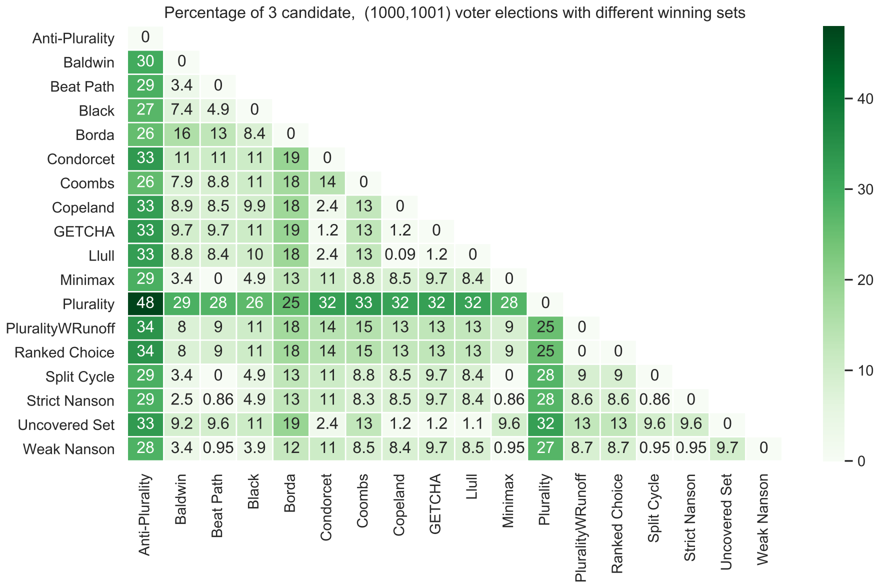
✓ respond in a reasonable way to **new candidates** joining the election (Stability for Winners, Immunity of Spoilers);

2. respond in a reasonable way to **new voters** joining the election.

	Split Cycle	Ranked Pairs	Beat Path	Mini-max	Copeland	Borda	Coombs	Instant Runoff	Plurality
Condorcet Winner	✓	✓	✓	✓	✓	–	–	–	–
Condorcet Loser	✓	✓	✓	–	✓	✓	✓	✓	–
Immunity to Spoilers	✓	–	–	✓	✓	–	–	–	–
Stability for Winners	✓	–	–	–	–	–	–	–	–

Quantitative Analysis of Voting Methods

- How often do different voting methods give different answers?
- What is the frequency of voting paradoxes?
- How should we use the frequency of voting paradoxes to compare voting methods?


Percentage of 3 candidate, (1000,1001) voter elections with different winning sets
Anti-Plurality 0
Baldwin 30 0
Beat Path 29 3.4 0
Black 27 7.4 4.9 0
Borda 26 16 13 8.4 0
Condorcet 33 11 11 11 19 0
Coombs 26 7.9 8.8 11 18 14 0
Copeland 33 8.9 8.5 9.9 18 2.4 13 0
GETCHA 33 9.7 9.7 11 19 1.2 13 1.2 0
Llull 33 8.8 8.4 10 18 2.4 13 0.09 1.2 0
Minimax 29 3.4 0 4.9 13 11 8.8 8.5 9.7 8.4 0
Plurality 48 29 28 26 25 32 33 32 32 32 28 0
PluralityWRunoff 34 8 9 11 18 14 15 13 13 13 9 25 0
Ranked Choice 34 8 9 11 18 14 15 13 13 13 9 25 0 0
Split Cycle 29 3.4 0 4.9 13 11 8.8 8.5 9.7 8.4 0 28 9 9 0
Strict Nanson 29 2.5 0.86 4.9 13 11 8.3 8.5 9.7 8.4 0.86 28 8.6 8.6 0.86 0
Uncovered Set 33 9.2 9.6 11 19 2.4 13 1.2 1.2 1.1 9.6 32 13 13 9.6 9.6 0
Weak Nanson 28 3.4 0.95 3.9 12 11 8.5 8.4 9.7 8.5 0.95 27 8.7 8.7 0.95 0.95 9.7 0
Anti-Plurality
Baldwin
Beat Path
Black
Borda
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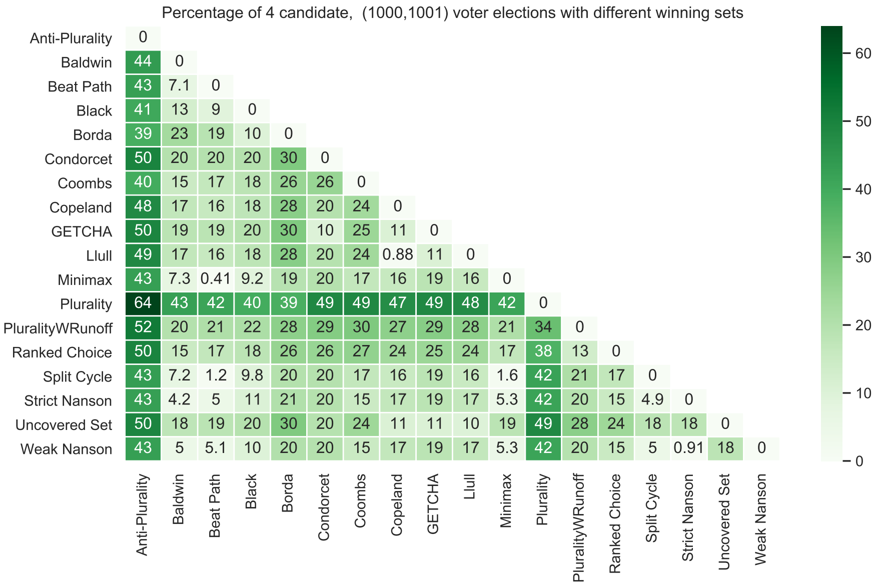
Minimax
Plurality
PluralityWRunoff
Ranked Choice
Split Cycle
Strict Nanson
Uncovered Set
Weak Nanson
40
30
20
10
0



Percentage of 4 candidate, (1000,1001) voter elections with different winning sets
Anti-Plurality 0
Baldwin 44 0
Beat Path 43 7.1 0
Black 41 13 9 0
Borda 39 23 19 10 0
Condorcet 50 20 20 20 30 0
Coombs 40 15 17 18 26 26 0
Copeland 48 17 16 18 28 20 24 0
GETCHA 50 19 19 20 30 10 25 11 0
Llull 49 17 16 18 28 20 24 0.88 11 0
Minimax 43 7.3 0.41 9.2 19 20 17 16 19 16 0
Plurality 64 43 42 40 39 49 49 47 49 48 42 0
PluralityWRunoff 52 20 21 22 28 29 30 27 29 28 21 34 0
Ranked Choice 50 15 17 18 26 26 27 24 25 24 17 38 13 0
Split Cycle 43 7.2 1.2 9.8 20 20 17 16 19 16 1.6 42 21 17 0
Strict Nanson 43 4.2 5 11 21 20 15 17 19 17 5.3 42 20 15 4.9 0
Uncovered Set 50 18 19 20 30 20 24 11 11 10 19 49 28 24 18 18 0
Weak Nanson 43 5 5.1 10 20 20 15 17 19 17 5.3 42 20 15 5 0.91 18 0
Anti-Plurality
Baldwin
Beat Path
Black
Borda
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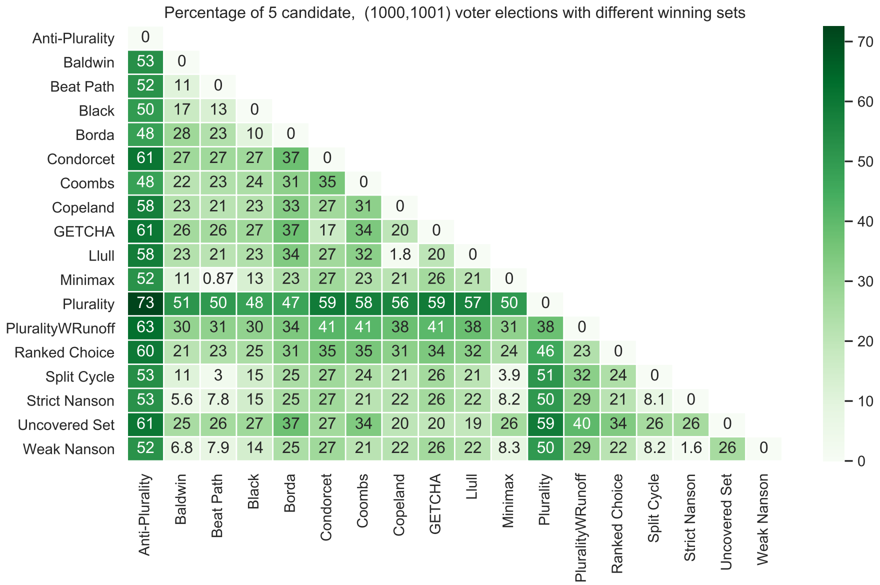
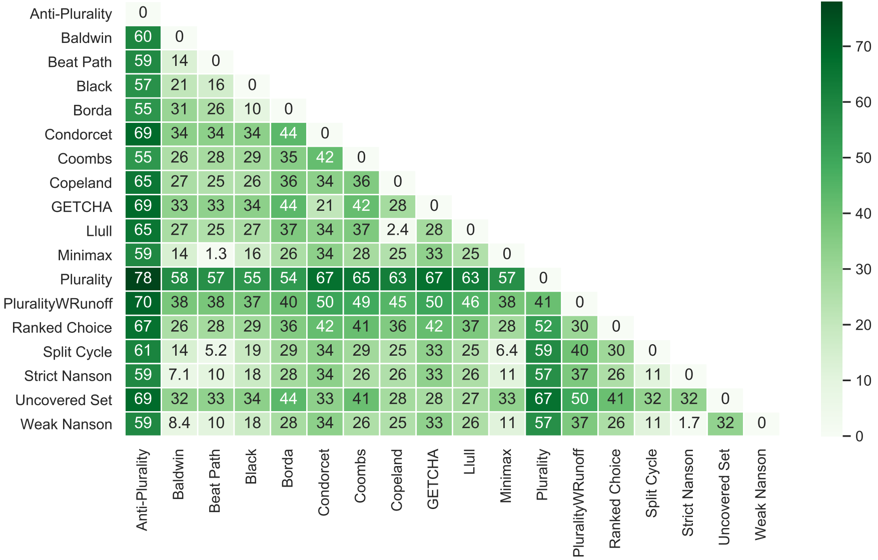
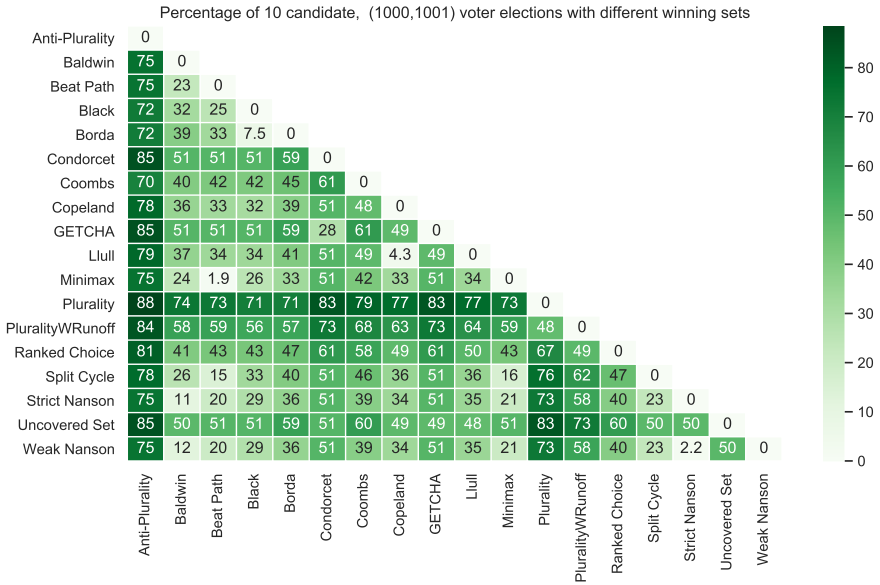
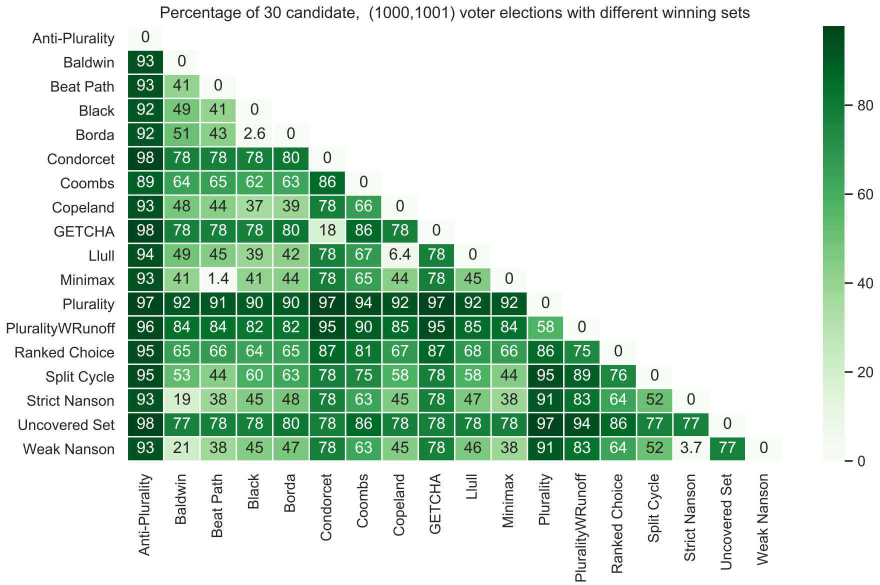
Copeland
GETCHA
Llull
Minimax
Plurality
PluralityWRunoff
Ranked Choice
Split Cycle
Strict Nanson
Uncovered Set
Weak Nanson
60
50
40
30
20
10
0


Percentage of 5 candidate, (1000,1001) voter elections with different winning sets
Anti-Plurality 0
Baldwin 53 0
Beat Path 52 11 0
Black 50 17 13 0
Borda 48 28 23 10 0
Condorcet 61 27 27 27 37 0
Coombs 48 22 23 24 31 35 0
Copeland 58 23 21 23 33 27 31 0
GETCHA 61 26 26 27 37 17 34 20 0
Llull 58 23 21 23 34 27 32 1.8 20 0
Minimax 52 11 0.87 13 23 27 23 21 26 21 0
Plurality 73 51 50 48 47 59 58 56 59 57 50 0
PluralityWRunoff 63 30 31 30 34 41 41 38 41 38 31 38 0
Ranked Choice 60 21 23 25 31 35 35 31 34 32 24 46 23 0
Split Cycle 53 11 3 15 25 27 24 21 26 21 3.9 51 32 24 0
Strict Nanson 53 5.6 7.8 15 25 27 21 22 26 22 8.2 50 29 21 8.1 0
Uncovered Set 61 25 26 27 37 27 34 20 20 19 26 59 40 34 26 26 0
Weak Nanson 52 6.8 7.9 14 25 27 21 22 26 22 8.3 50 29 22 8.2 1.6 26 0
Anti-Plurality
Baldwin
Beat Path
Black
Borda
Condorcet
Coombs
Copeland
GETCHA
Llull
Minimax
Plurality
PluralityWRunoff
Ranked Choice
Split Cycle
Strict Nanson
Uncovered Set
Weak Nanson
70
60
50
40
30
20
10
0

Percentage of 6 candidate, (1000,1001) voter elections with different winning sets

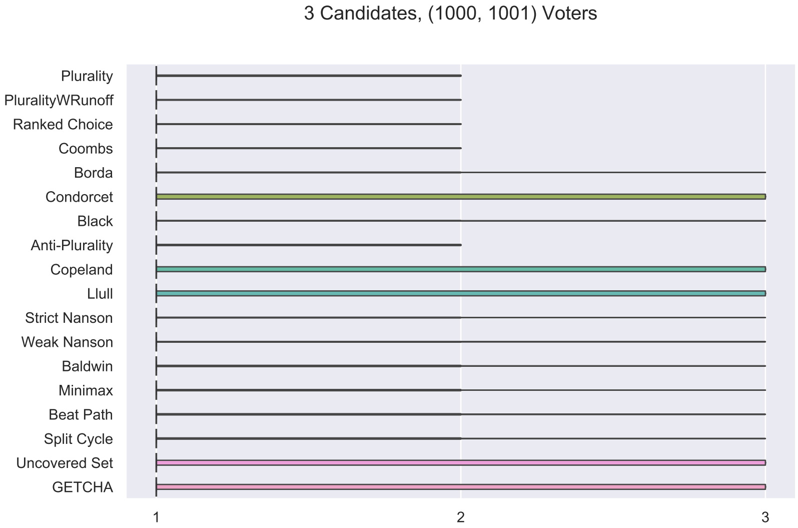



Percentage of 10 candidate, (1000,1001) voter elections with different winning sets
Anti-Plurality 0
Baldwin 75 0
Beat Path 75 23 0
Black 72 32 25 0
Borda 72 39 33 7.5 0
Condorcet 85 51 51 51 59 0
Coombs 70 40 42 42 45 61 0
Copeland 78 36 33 32 39 51 48 0
GETCHA 85 51 51 51 59 28 61 49 0
Llull 79 37 34 34 41 51 49 4.3 49 0
Minimax 75 24 1.9 26 33 51 42 33 51 34 0
Plurality 88 74 73 71 71 83 79 77 83 77 73 0
PluralityWRunoff 84 58 59 56 57 73 68 63 73 64 59 48 0
Ranked Choice 81 41 43 43 47 61 58 49 61 50 43 67 49 0
Split Cycle 78 26 15 33 40 51 46 36 51 36 16 76 62 47 0
Strict Nanson 75 11 20 29 36 51 39 34 51 35 21 73 58 40 23 0
Uncovered Set 85 50 51 51 59 51 60 49 49 48 51 83 73 60 50 50 0
Weak Nanson 75 12 20 29 36 51 39 34 51 35 21 73 58 40 23 2.2 50 0
Anti-Plurality
Baldwin
Beat Path
Black
Borda
Condorcet
Coombs
Copeland
GETCHA
Llull
Minimax
Plurality
PluralityWRunoff
Ranked Choice
Split Cycle
Strict Nanson
Uncovered Set
Weak Nanson
80
70
60
50
40
30
20
10
0



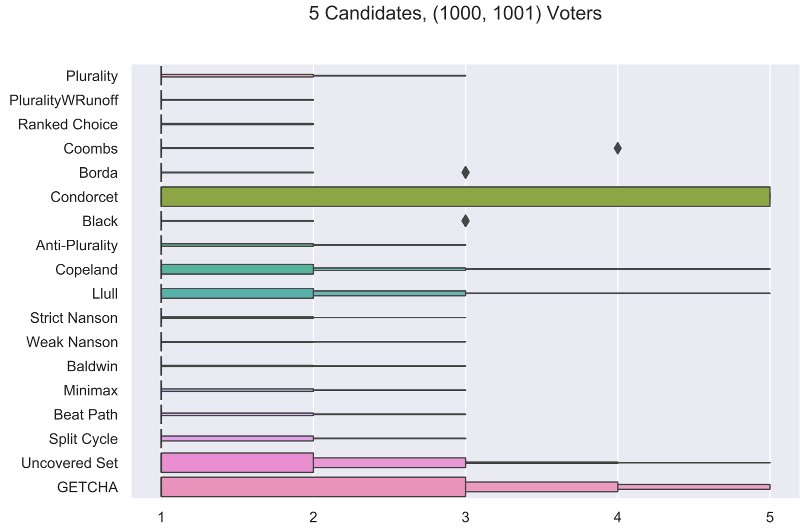
	Anti-Plurality	Baldwin	Beat Path	Black	Borda	Condorcet	Coombs	Copeland	GETCHA	Llull	Minimax	Plurality	PluralityWRunoff	Ranked Choice	Split Cycle	Strict Nanson	Uncovered Set	Weak Nanson
Anti-Plurality	0																	
Baldwin	93	0																
Beat Path	93	41	0															
Black	92	49	41	0														
Borda	92	51	43	2.6	0													
Condorcet	98	78	78	78	80	0												
Coombs	89	64	65	62	63	86	0											
Copeland	93	48	44	37	39	78	66	0										
GETCHA	98	78	78	78	80	18	86	78	0									
Llull	94	49	45	39	42	78	67	6.4	78	0								
Minimax	93	41	1.4	41	44	78	65	44	78	45	0							
Plurality	97	92	91	90	90	97	94	92	97	92	92	0						
PluralityWRunoff	96	84	84	82	82	95	90	85	95	85	84	58	0					
Ranked Choice	95	65	66	64	65	87	81	67	87	68	66	86	75	0				
Split Cycle	95	53	44	60	63	78	75	58	78	58	44	95	89	76	0			
Strict Nanson	93	19	38	45	48	78	63	45	78	47	38	91	83	64	52	0		
Uncovered Set	98	77	78	78	80	78	86	78	78	78	78	97	94	86	77	77	0	
Weak Nanson	93	21	38	45	47	78	63	45	78	46	38	91	83	64	52	3.7	77	0

Sizes of Winning Sets



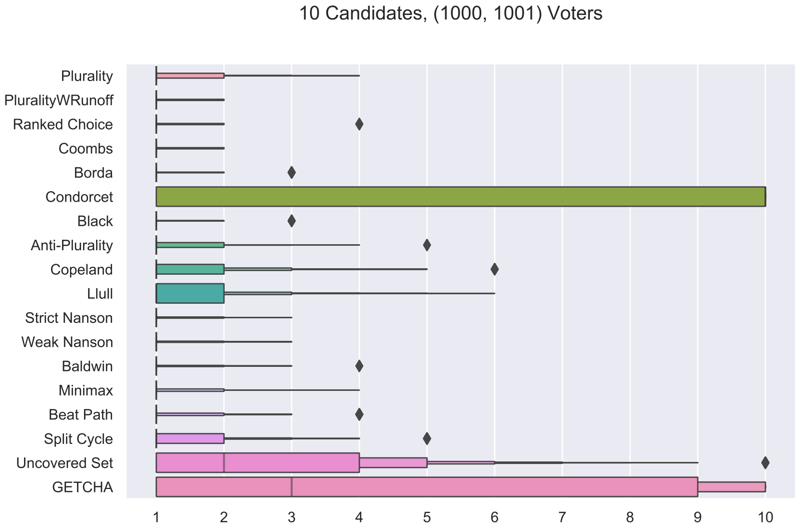
Sizes of Winning Sets

Sizes of Winning Sets

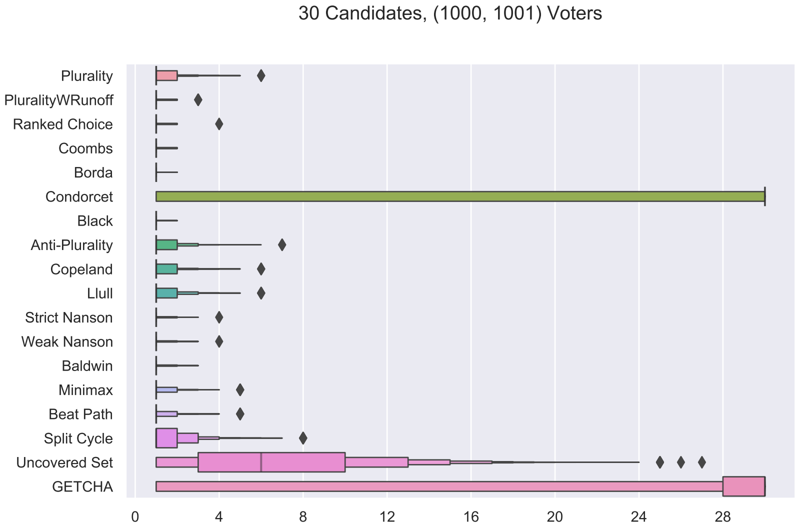


Sizes of Winning Sets

Sizes of Winning Sets



Sizes of Winning Sets



Models of voters behavior: IC (Impartial culture), IAC (Impartial anonymous culture), IANC (Impartial anonymous and neutral culture), Mallows models, Spatial models, Structured Preferences (e.g., Single Peaked models)

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