Tools for Formal Epistemology: Doxastic Logic, Probability and Default Logic

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Lecture 3

ESSLLI 2023

$$p_0 \quad \overset{\text{Learn that } E}{\Longrightarrow} \quad p_t(\cdot) = p_0(\cdot \mid E)$$

Conditional Probability

The probability of E *given* F, dented $p(E|F)$, is defined to be

$$p(E|F) = \frac{p(E \cap F)}{p(F)}.$$

provided $P(F) > 0$.

Setting $p_t(\cdot) = p_0(\cdot \mid E)$ is demonstrably the correct thing to do just in case, for all propositions $H \in \Sigma$, both:

1. Certainty: $p_t(E) = 1$
2. Rigidity: $p_t(H \mid E) = p_0(H \mid E)$

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Is there a proposition E such that $p_t(\cdot) = p_0(\cdot \mid E)$?

Jeffrey Conditionalization

When an observation bears directly on the probabilities over a partition $\{E_i\}$, changing them from $p(E_i)$ to $q(E_i)$, the new probability for any proposition H should be

$$q(H) = \sum_i p(H \mid E_i) q(E_i)$$

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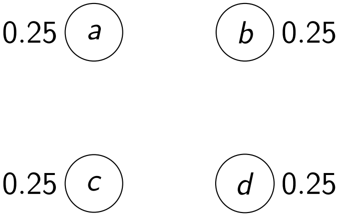
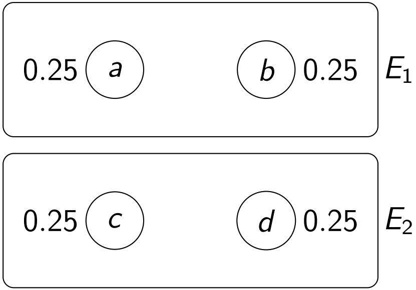
Fact: If q is obtained from p by Jeffrey Conditioning on the partition $\{E, \overline{E}\}$ with $q(E) = 1$, then $q(\cdot) = p(\cdot \mid E)$.

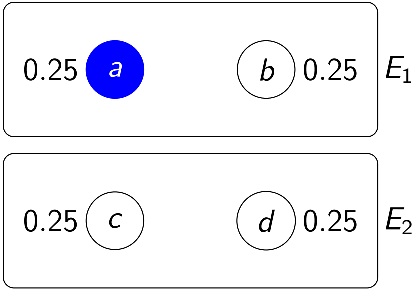

0.25
a
b
0.25
0.25
c
d
0.25



The probability that the guilty party is left-handed is 0.8

$$E_1 = \{a, b\},\ E_2 = \{c, d\}$$

$$p(E_1) = 0.8 \qquad p(E_2) = 0.2$$



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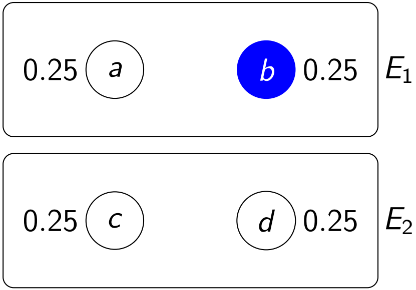
$$p(a) = p_0(\{a\} \mid E_1) * p(E_1) + p_0(\{a\} \mid E_2) * p(E_2) = 0.5 * 0.8 + 0 = 0.4$$



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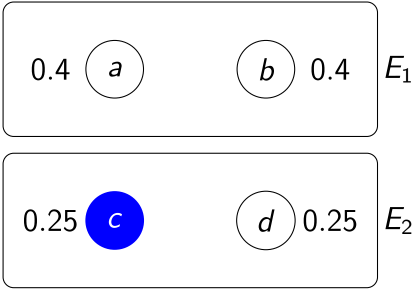
$E_1 = \{a, b\}, \; E_2 = \{c, d\}$

$p(E_1) = 0.8 \qquad p(E_2) = 0.2$

$p(b) = p_0(\{b\} \mid E_1) * p(E_1) + p_0(\{b\} \mid E_2) * p(E_2) = 0 + 0.5 * 0.8 = 0.4$



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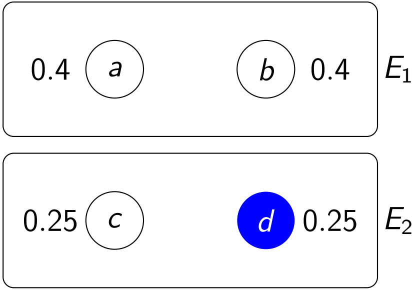
$$p(E_1) = 0.8 \qquad p(E_2) = 0.2$$

$$p(c) = p_0(\{c\} \mid E_1) * p(E_1) + p_0(\{c\} \mid E_2) * p(E_2) = 0 + 0.5 * 0.2 = 0.1$$



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$$E_1 = \{a, b\},\ E_2 = \{c, d\}$$

$$p(E_1) = 0.8 \qquad p(E_2) = 0.2$$

$$p(d) = p_0(\{d\} \mid E_1) * p(E_1) + p_0(\{d\} \mid E_2) * p(E_2) = 0 + 0.5 * 0.2 = 0.1$$

P. Diaconis and S. Zabell. *Updating Subjective Probability*. Journal of the American Statistical Association, Vol. 77, No. 380., pp. 822-830 (1982).

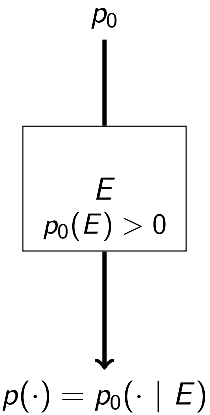
Fact. Jeffrey conditioning is not commutative.

Commutativity on Experiences Any rule for updating degrees of belief on experiences should be such that the result of updating credences on one experience and then another should be the same as the result of updating on the same two experiences in reverse order.

J. Weisberg. *Commutativity or Holism? A Dilemma for Conditionalizers*. British Journal of the Philosophy of Science, 60(4), pp. 793-812, 2009.

M. Lange. *Is Jeffrey Conditionalization Defective in Virtue of Being NonCommutative? Remarks on the Sameness of Sensory Experience*. Synthese 123: 393-403, 2000.

C. Wagner. *Probability kinematics and commutativity*. Philosophy of Science 69, 266-278, 2002.

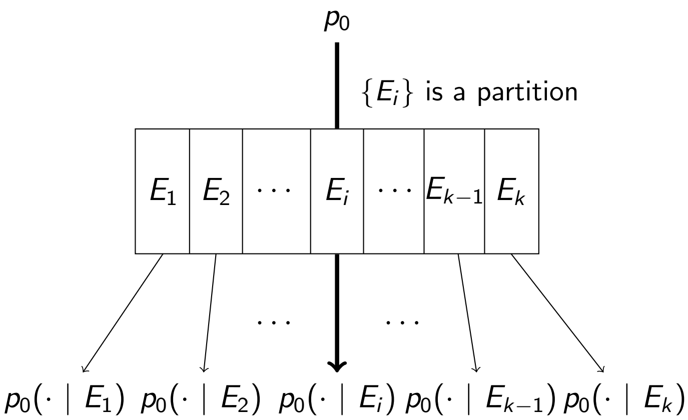

p_0
E
$p_0(E) > 0$
$p(\cdot) = p_0(\cdot \mid E)$

$$p_0$$

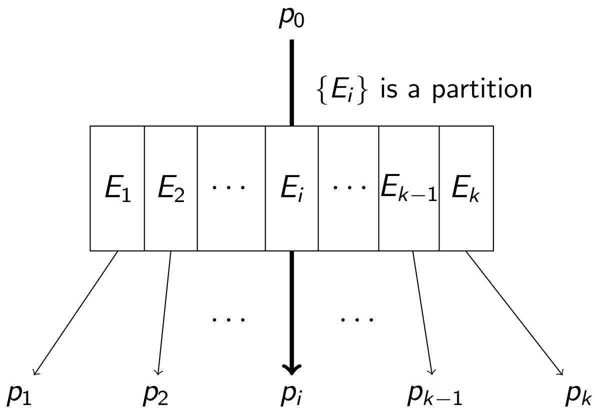
$$(E_1 : q_1, \ldots, E_k : q_k)$$

$\{E_i\}$ is a partition, $\sum_i q_i = 1$

$$p(\cdot) = \sum_i q_i * p_0(\cdot \mid E_i)$$

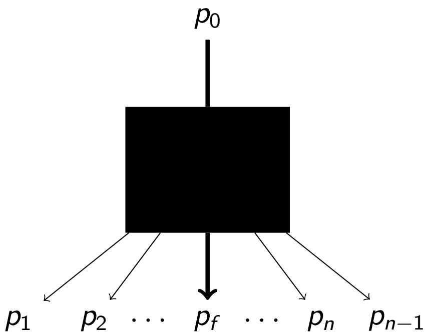


p_0
$\{E_i\}$ is a partition
E_1 E_2 $\cdots$ E_i $\cdots$ E_{k-1} E_k
$\cdots$ $\cdots$
$p_0(\cdot \mid E_1)$ $p_0(\cdot \mid E_2)$ $p_0(\cdot \mid E_i)$ $p_0(\cdot \mid E_{k-1})$ $p_0(\cdot \mid E_k)$

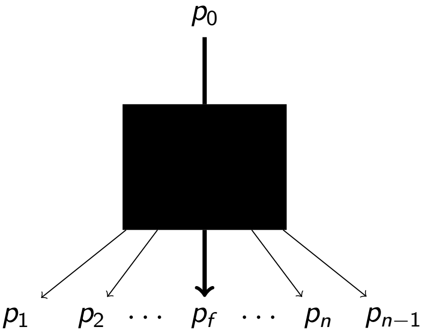

p_0
$\{E_i\}$ is a partition
E_1
E_2
$\cdots$
E_i
$\cdots$
E_{k-1}
E_k
$\cdots$
$\cdots$
p_1
p_2
p_i
p_{k-1}
p_k

Suppose that you are in a learning situation even more amorphous than the kind which motivates Jeffrey's idea. There is no nontrivial partition that you expect with probability one to be sufficient for your belief change....Perhaps you are in a novel situation where you expect the unexpected observational input....You are going to just think about some subject matter and update as a result of your thoughts...I will consider the learning situation a kind of black box and attempt no analysis of its internal structure.

(Skyrms, pg. 96, 97)



p_0
p_1
p_2
$\cdots$
p_f
$\cdots$
p_n
p_{n-1}



(Reflection/Martingale Property) $\quad p_0(A \mid p_f) = p_f(A)$

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Suppose that your anticipated future degree of belief for A is given by a random variable X. So, for instance, $X = r$ and $r < X < s$ can be assigned probabilities.

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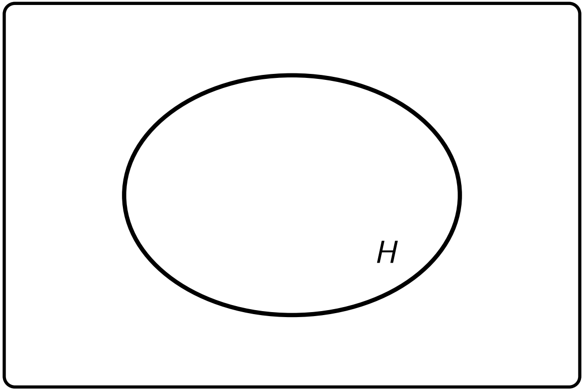
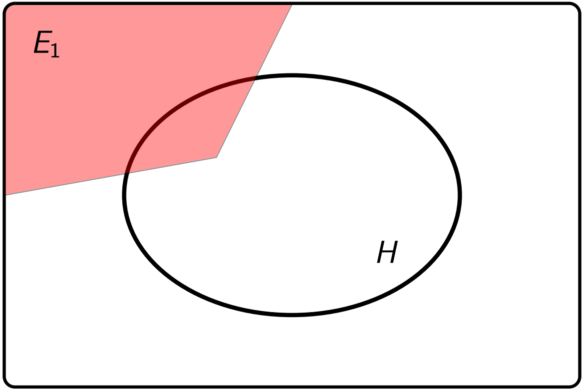
$p_0(A) = \sum_r p_0(A \mid X = r) p_0(X = r)$

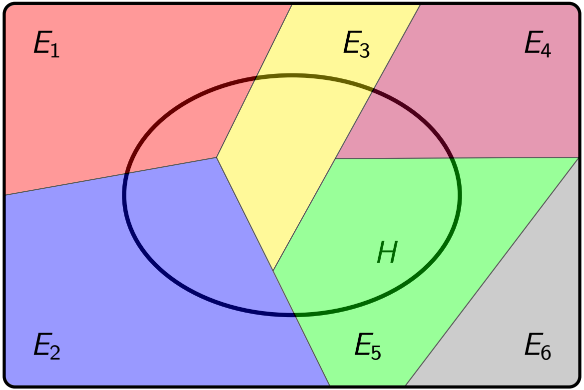
A basic result about probabilities.

For any finite partition $\{E_i\}$ of the state space and any event H,

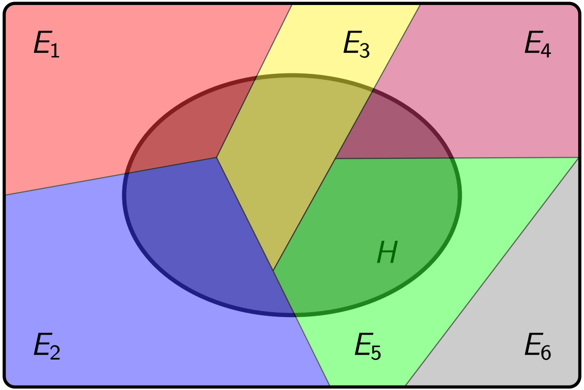
$$p(H) = \sum_i p(E_i) p(H \mid E_i)$$


H

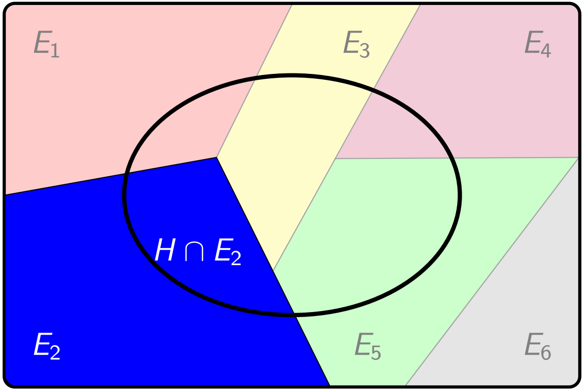

E_1
H



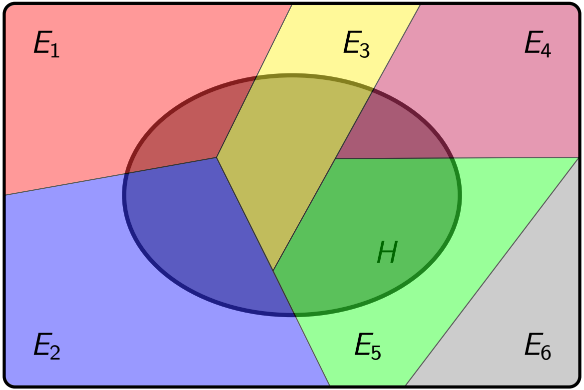
E_1
E_3
E_4
H
E_2
E_5
E_6



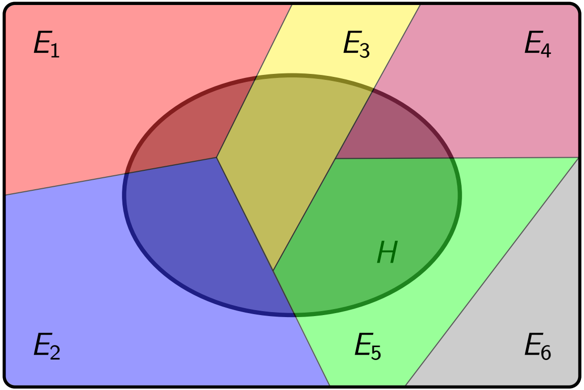
$$p(H) = p(H \cap E_1) + \cdots + p(H \cap E_6)$$



$$p(H) \quad = \quad p(H \cap E_1) + p(H \cap E_2) + \cdots + p(H \cap E_6)$$



$$
\begin{aligned}
p(H) &= p(H \cap E_1) + \cdots + p(H \cap E_6) \\
&= \frac{p(E_1)}{p(E_1)} p(H \cap E_1) + \cdots + \frac{p(E_6)}{p(E_6)} p(H \cap E_6)
\end{aligned}
$$



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&= \textstyle\sum_i p(E_i) P(H \mid E_i)
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Suppose that $p_0(A \mid E_k) \neq p_0(A \mid E_j)$ for any $E_k, E_j \in \mathfrak{P}$. Then, $p_0(A \mid p_0(A \mid \mathfrak{P}) = r) = p_0(A \mid E)$ where $E \in \mathfrak{P}$ is the unique element with $p_0(A \mid E) = r$. So, $p_0(A \mid p_0(A \mid \mathfrak{P}) = r) = r$.

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Suppose that $E_1, \ldots, E_m$ are the elements of $\mathfrak{P}$ such that $p_0(A \mid E_i) = r$ for all $1 \leq i \leq m$. Then, $p_0(A \mid E_1 \cup \cdots \cup E_m) = r$. Note that $p_0(A \mid p_0(A \mid \mathfrak{P}) = r) = p_0(A \mid E_1 \cup \cdots \cup E_m) = r$.

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Huttegger develops an account in which the reflection principle is a necessary condition for a black-box probability update to count as a *genuine learning experience*.

Simon Huttegger. *Learning Experiences and the Value of Knowledge*. Philosophical Studies, 2013.

The Value of Knowledge

Why is it better to make a "more informed" decision?
Suppose that you can either choose know, or perform a costless experiment and make the decision later. What should you do?

I. J. Good. *On the principle of total evidence*. British Journal for the Philosophy of Science, 17, pgs. 319 - 321, 1967.

"Never decide today what you might postpone until tomorrow in order to learn something new"

Choose between n acts $A_1, \ldots, A_n$ (with states K_i) or perform a cost-free experiment with possible results $\{e_k\}$, then decide.

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Then,

$$U(\text{Choose now}) = \max_j \sum_i p(K_i) U(A_j \mathbin{\&} K_i)$$

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Bayes Theorem. $p(K_i|e_j) = p(e_j|K_i)\frac{p(K_i)}{p(e_j)}$

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The value of an informed decision conditional on e:

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Compare $\max_j \sum_k \sum_i p(K_i) p(e_k \mid K_i) U(A_j \,\&\, K_i)$ and $\sum_k \max_j \sum_i p(e_k \mid K_i) p(K_i) U(A_j \,\&\, K_i)$

The value of an informed decision conditional on e:

$$\max_j \sum_i p(K_i \mid e) U(A_j \ \& \ K_i)$$

$$\begin{aligned} U(\text{Learn, Choose }) &= \textstyle\sum_k p(e_k) \max_j \sum_i p(K_i \mid e_k) U(A_j \ \& \ K_i) \\ &= \textstyle\sum_k p(e_k) \max_j \sum_i \left(\frac{p(e_k \mid K_i) p(K_i)}{p(e_k)}\right) U(A_j \ \& \ K_i) \\ &= \textstyle\sum_k \max_j \sum_i p(e_k \mid K_i) p(K_i) U(A_j \ \& \ K_i) \end{aligned}$$

Compare $\max_j \sum_k \sum_i p(K_i) p(e_k \mid K_i) U(A_j \ \& \ K_i)$ and $\sum_k \max_j \sum_i p(e_k \mid K_i) p(K_i) U(A_j \ \& \ K_i)$

$\sum_k \max_j g(k, j)$ is greater than or equal to $\max_j \sum_k g(k, j)$, so the second is greater than or equal to the first.

(M) implies value of knowledge

Suppose that (M) holds.

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$$= \max_j \sum_f \sum_i p_f(K_i)p(p_f)u(A_j \& K_i)$$

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The value of choosing after the learning experience is:

$$\sum_f p(p_f) \max_j \sum_i p_f(K_i)u(A_j\&K_i)$$

(M) implies value of knowledge

Suppose that (M) holds.
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The value of choosing after the learning experience is:

$$\sum_f p(p_f) \max_j \sum_i p_f(K_i)u(A_j \& K_i)$$

The latter term cannot be less than the former term on general mathematical grounds.

- The experiment is assumed to be essentially costless;
- You know that you are an expected utility maximizer and that you will be one after learning the true member of the partition.
- In the classical theorem you know that you will update by conditioning; in Skyrms' extension, you know that you will honor the martingale principle.
- By working within Savage's decision theory, the states and acts are probabilistically independent (choosing an act does not give any information about the state).

- The states, acts and utilities are the same before and after the learning experience.

- Having the learning experience does not by itself alter your probabilities for states of the world (although the outcomes of the experience usually do); the learning experience and the states of the world are probabilistically independent.

...the martingale principle should not be applied to belief changes in epistemologically defective situations. In situations of memory loss, of being brainwashed or being under the influence of drugs, (M) should obviously not hold. If you believe that in an hour you will think you can fly because you're about to consume some funny looking pills, then you should not already now have that belief.

So, the martingale principle is claimed to apply if you learn something in the black-box, but not if you learn nothing or other things happen besides learning.

A genuine learning situation is partially characterized in the following way:

Postulate. If a belief change from p to $\{pf\}$ constitutes a genuine learning situation, then

$$\sum_f p(p_f) \max_j \sum_i p_f(K_i)u(A_j \& K_i) \geq \max \sum_i p(K_i)u(A_j \& K_i)$$

for all utility values $u(A_j \& K_i)$ with strict inequality unless the same act maximizes expected utility irrespective of which of the p_f occurs.

If a belief change leads you to foreseeably make worse choices than you could already make now in some decision situations, then it cannot be a pure learning experience. Perhaps you are bolder after having taken those funny looking pills, for example. From your current perspective, this might help you in some decision problems, but it will be harmful in others.

J. Kadane, M. Schervish and T. Seidenfeld. *Is Ignorance Bliss?* . The Journal of Philosophy, 105, pp. 5-36, 2008.

Nilanjan Das. *The Value of Biased Information*. The British Journal for the Philosophy of Science, 74(1), 2023.

Simon Huttegger and Michael Nielsen. *Generalized Learning and Conditional Expectation*. Philosophy of Science 87 (5), pp. 868-883, 2020.

Tools for Formal Epistemology:
Doxastic Logic, Probability, and Default Logic
– ESSLLI 2023 –

Lecture 3.2 – Beginning Default Logic

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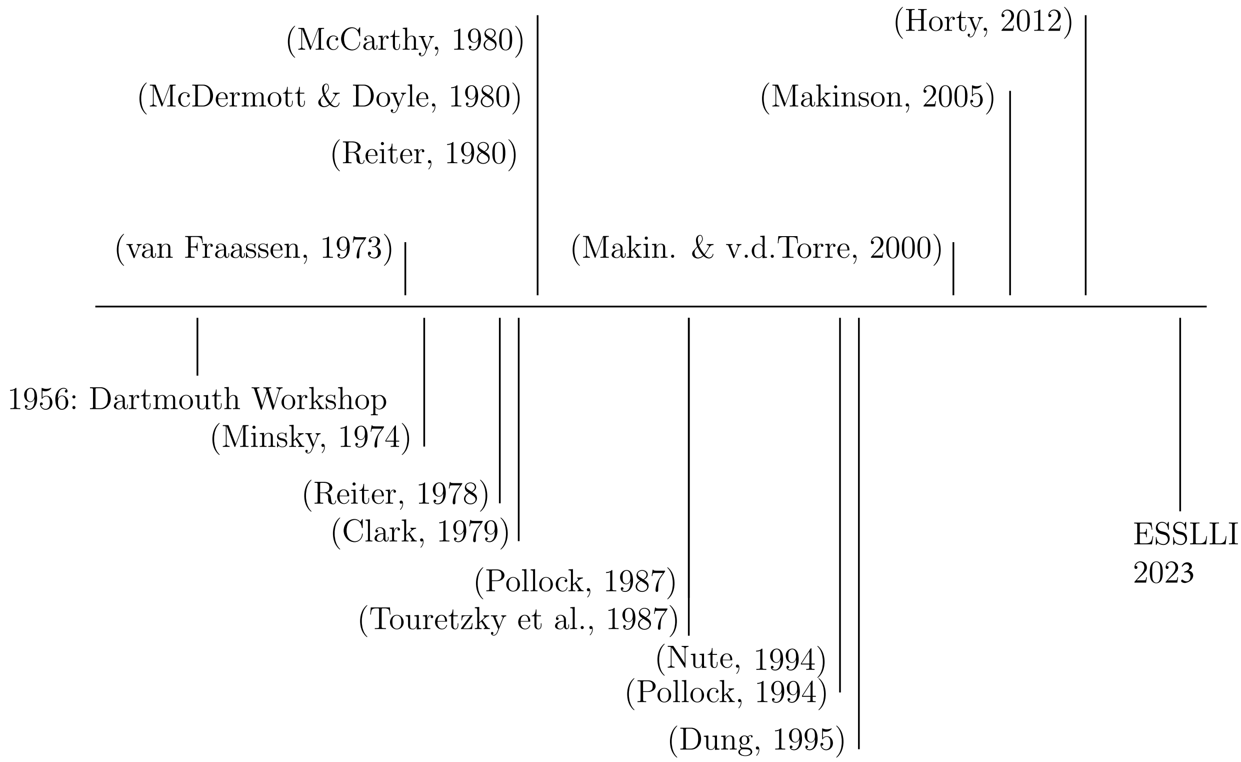
1 Unpacking the course title

- What is epistemology? What is *formal* epistemology?
- Toolbox
- How formal does formal epistemology need to be?
- The remainder of the course (cf. ESSLLI33):
  - default logic (Wed / Thu)
  - some applications in epistemology (Thu / Fri)

2 Default logic and defeasible (a.k.a. nonmonotonic) logics

- Monotonic consequence relation: if X follows from Γ, then X follows from $\Gamma \cup \Delta$.
  Of course, classical logic is monotonic.
- Defeasible (or nonmonotonic) logics are, roughly, formal systems with a consequence relations that are not monotonic, often intended to capture our commonsense reasoning.
  For some Γ, Δ, X: $\Gamma \mid\!\sim X$, but $\Gamma \cup \Delta \not\mid\!\sim X$.
- Default logic is one kind of defeasible logic.



3 Why default logic (and not some other system)?

- An early historically important system
- Good balance between accessibility and expressive power
- Appears to be particularly good for modeling reasoning
  (more) explicitly

4 Default theories

4.1 Notation

- Propositions: A, B, C, $\ldots$, $\top$
- Connectives: $\wedge$, $\vee$, $\neg$, $\supset$
- Classical consequence: $\vdash$
- Logical closure: $Th(\mathcal{E}) = \{A : \mathcal{E} \vdash A\}$

4.2 Standard example

- Tweety is a bird
  Therefore, Tweety is able to fly

Why? Well, bird typically can fly!

- Tweety is a bird
  Tweety is a penguin
  Therefore, Tweety is not able to fly

Why? Well, penguins typically can't fly!

4.3 Default rules

- Rules of the form $X \to Y$
  Example: $B(t) \to F(t)$
  Instance of: $B(x) \to F(x)$ ("Birds fly")
  Given our language, simply $B \to F$

4.4 Premise and conclusion

- Where $\delta = X \to Y$, let
  $Premise(\delta) = X$
  $Conclusion(\delta) = Y$
- Where $\mathcal{S}$ is a set of defaults, let
  $Conclusion(\mathcal{S}) = \{Conclusion(\delta) : \delta \in \mathcal{S}\}$.

4.5 Priority ordering on defaults

- $\delta < \delta'$ says that δ' is stronger than δ
  $<$ is a *strict partial order*, or an irreflexive and transitive relation
- Priorities can have various sources:
  Specificity
  Reliability
  Authority
  Relative confidence
  ...
  (In general, this is going to depend on the application)

4.6 Default theories

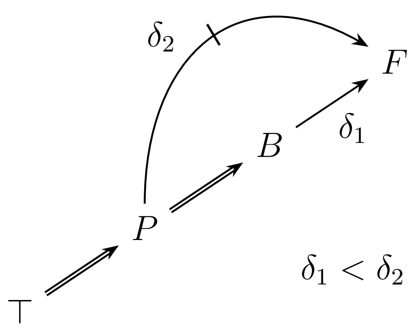
- A default theory is a tuple $\langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
  $\mathcal{W}$ is a set of propositional formulas
  $\mathcal{D}$ is a set of default rules
  and $<$ is an ordering on $\mathcal{D}$

4.7 Example (Tweety Triangle)

- $\Delta_1 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where
  $\mathcal{W} = \{P, P \supset B\}$
  $\mathcal{D} = \{\delta_1, \delta_2\}$
  - $\delta_1 = B \to F$
  - $\delta_2 = P \to \neg F$

  $\delta_1 < \delta_2$
  (P = Tweety is a penguin, B = Tweety is a bird, F = Tweety can fly)

4.8 Inference graphs



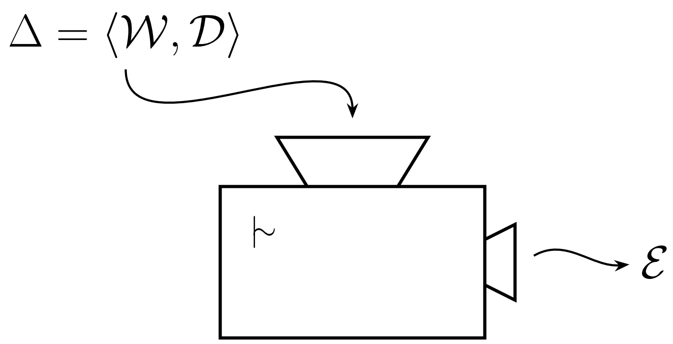
5 From default theories to extensions

5.1 Core question (of default logic)

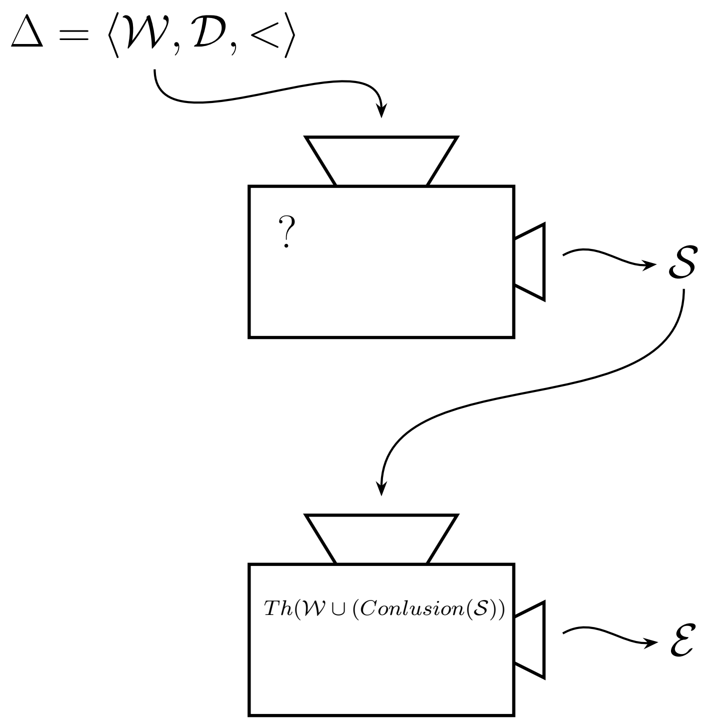
- What can one conclude from a default theory?
  Alternatively: What is the *extension* of a given default theory $\langle \mathcal{W}, \mathcal{D}, < \rangle$?
- extension = a belief set an ideal reasoner might arrive at, based on the information contained in $\langle \mathcal{W}, \mathcal{D}, < \rangle$

5.2 Reiter vs. Horty

- Reiter defines extensions directly (and many others do too):



- Horty (2012) takes a roundabout way
- A *scenario* based on $\langle \mathcal{W}, \mathcal{D}, < \rangle$ is any subset $\mathcal{S}$ of the defaults $\mathcal{D}$
- A *proper scenario* is the (intuitively) "right" subset of $\mathcal{D}$
- An *extension* $\mathcal{E}$ based on $\langle \mathcal{W}, \mathcal{D}, < \rangle$, then, turns into $\mathcal{E} = Th(\mathcal{W} \cup Conclusion(\mathcal{S}))$, where $\mathcal{S}$ is proper.



5.3 Example (Tweety)

- $\Delta_1 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where
  $\mathcal{W} = \{P, P \supset B\}$
  $\mathcal{D} = \{\delta_1, \delta_2\}$
  - $\delta_1 = B \to F$
  - $\delta_2 = P \to \neg F$

  $\delta_1 < \delta_2$
- You're given Δ_1.. What are the scenarios based on it?
- which of these is the proper one?
- what is the extension?

5.4 Next step

- Specify how to select proper scenarios..

6 Binding defaults

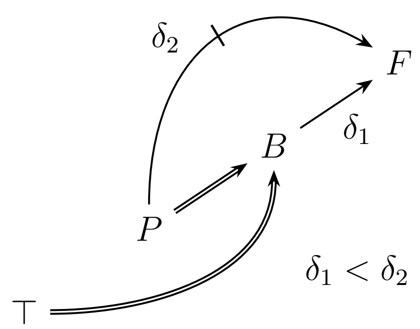
6.1 Intuition

- *Binding defaults* are, intuitively, the *good* or *correct* ones to base the conclusions on in the context. Proper scenarios will contain all and only such defaults.
- The concept will emerge as a combination of three preliminary notions: (i) *triggering*, (ii) *conflict*, and (iii) *defeat*.

6.2 Triggered defaults

- $Triggered_{\langle \mathcal{W}, \mathcal{D}, < \rangle}(\mathcal{S}) = \{\delta \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash Premise(\delta)\}$

6.3 Example (Tweety)



- $\Delta_1' = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where
  $\mathcal{W} = \{B, P \supset B\}$
  $\mathcal{D} = \{\delta_1, \delta_2\}$

- $\delta_1 = B \rightarrow F$
- $\delta_2 = P \rightarrow \neg F$

$\delta_1 < \delta_2$

- Here $Triggered_{\Delta_1}(\emptyset) = \{\delta_1\}$

$\ldots$