PHIL 470 - Logical Theory II
Incompleteness and Undecidability

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Semester:	Spring 2024
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Office Hours:	TBA
Class Times:	TuTh 3:30pm - 4:45pm
Class Location:	SQH 2120

Course Description

This course will focus on Kurt Gödel's first and second incompleteness theorems. The first incompleteness theorem states, roughly, that every logical system that is sufficiently expressive and free from contradictions is incomplete in the sense that there are always statements such that neither the statement nor its negation can be proved. The second incompleteness theorem states that sufficiently expressive arithmetic theories cannot prove their own consistency. We will prove the 1st and 2nd Incompleteness Theorems and survey their technical and philosophical repercussions.

The primary goal of the course is to introduce the technical and philosophical topics that arise when proving Gödel's Incompleteness Theorems. Topics to be covered include: formal models of computation (especially elementary recursion theory); the Church-Turing Thesis; Gödel's 1st and 2nd incompleteness theorems and their repercussions; Tarski's proof of the undefinability of truth; Undecidability of the Halting Problem; provability logic - Kripke soundness and completeness (time permitting), absolute provability and The Knower Paradox (time permitting); and non-standard models of arithmetic (time permitting).

Prerequisites: PHIL370 (or equivalent logic course), or permission from the instructor.

Required Resources

- Course Website: `https://umd.instructure.com/courses/1362235`
- PollEverywhere: We will have short quizzes during each lecture administered through PollEverywhere. Please register at PollEverywhere for this course at `https://PollEv.com/epacuit/register?group_key=tI3f06ol4p16ucFq57RGRphOK`. The registration is free. *Make sure to use the same first and last name that is used in the ELMS gradebook.*
- This term we will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from both me and your classmates. Rather than emailing questions to me, I encourage you to post your questions on Piazza.

Reading

The readings for the course will be made available on the course website. Readings will be taken from the following texts:

1. *Incompleteness and Undecidability.* This text is compiled from the Open Logic Project: (openlogicproject.org).
2. G. Boolos, J. Burgess, and R. Jeffrey, *Computability and Logic*, Cambridge, 2007.
3. J. D. Hamkins, *Lectures on the Philosophy of Mathematics*, the MIT press, 2021.
4. P. Smith, *An Introduction to Gödel's Theorems*, Second Edition, Cambridge, 2020. `https://www.logicmatters.net/resources/pdfs/godelbook/GodelBookLM.pdf`.
5. P. Raatikainen, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy, https://plato.stanford.edu/entries/goedel-incompleteness/.

The following texts are recommended for additional reading:

- I. Chiswell and W. Hodges, *Mathematical Logic*, Oxford University Press, 2007.
- H.-D. Ebbinghaus, J. Flum, and W. Thomas, *Mathematical Logic*, Springer, 1995.
- H. Enderton, *A Mathematical Introduction to Logic*, Academic Press, 2nd Edition, 2001.
- M. Fitting, *Incompleteness in the Land of Sets*, Kings College Press, 2007.
- T. Franzen, *Gödel's Theorem: An Incomplete Guide to its Use and Abuse*, A K Peters, 2005.
- R. Smullyan, *Gödel's Incompleteness Theorems*, Oxford University Press, 1992.

Grading Policy

The course requirements are:

- Participation (50%): There are two parts for this grade:

    1. Short Quizzes: There will be a number of short quizzes given during each lecture. Typically, these short quizzes will consist of 1 question that is given during class using PollEverywhere, but some lectures may have more than 1 question. **Make-up quizzes will not be offered.** I will drop the lowest 5-10% of the quizzes (so you can miss some of the questions without losing any points).
    2. Questions: Students will be graded out of 10 points based on questions and comments asked during the semester. Your score will be based on either actively participating and asking questions during the lectures, or by asking questions or making comments on Piazza about the course material.

    This is a lecture course and we have a lot of material to cover, so you will easily fall behind if you miss lectures. You are responsible for learning any material covered in lectures that you miss. Please email me if you are going to miss class.

- Problem sets (40%): A number of different problem sets will be assigned throughout the semester. There will be a single grade for the problem sets which will be worth 100 points. There are three types of problem sets that will be assigned during the semester: Basis problems, Intermediate problems and challenge problems.

  - **Basic problems** (70% of your Problem Set score): You can work at your own pace. The due date for these problem sets will be the last day of class, **May 10**. However, you will lose 10 points if you do not submit these problem sets before the due date. You will have unlimited chances to correct your answers to these problems. I will grade partially completed problem sets.
  - **Intermediate problems** (20% of your Problem Set score): There will be 3-4 intermediate problem sets assigned during the semester.
  - **Challenge problems** (10% of your Problem Set score): There will be 1-2 challenge problem sets assigned towards the end of the semester.

  For the intermediate and challenge problem sets, you should attempt to solve as many of the problems as you can, but you do not necessarily need to solve all the problems. You should think of this as a writing assignment. The goal is to not only come up the correct answer to the questions, but present the answers clearly (indeed, for some problems I may happily provide you with the key idea to the solution). You are encouraged to work in small groups and to discuss your solutions with me. *But you must always do the final write-up completely on you own.*

- Final exam (10%). The final exam will be an in-class exam given during exam week. A review sheet containing the material that will be covered on the exam will be provided a week or two in advance of the exam.

Tentative Syllabus

Below is a tentative schedule for the semester (consult the course website for more details).

1. Introduction to Incompleteness (1 week)
2. Background: Sets, functions, relations, enumerations, mathematical induction (1 week)
3. Background: first order logic, natural deduction (1 week)
4. Formal Arithmetic (1 week)
5. Computability, Primitive Recursive Functions, Recursive Functions (2 weeks)
6. Arithmetization of syntax (Gödel numbering) (1 week)
7. Representability in Q (2 weeks)
8. The Diagonal Lemma and the First Incompleteness Theorem (1-2 week)
9. The Second Incompleteness Theorem (1 week)

10. Derivability conditions and Löb's Theorem (1-2 weeks)

Time permitting we will cover some of the following topics (depending on student interest).

1. Gödel's Disjunction, Absolute Provability and Epistemic Arithmetic, Montague and Kaplan's Knower Paradox
2. Models of arithmetic
3. Provability Logic
4. Second-order arithmetic

Important Dates

- Spring Break: March 17 - 24, No class on Tuesday, March 19 and Thursday, March 21
- Last Day of Class: Thursday May 9
- Exam Week: May 11 - 17 (the date of the exam will be announced later in the semester when it is scheduled).

Learning Outcomes

Students who successfully complete this course will be prepared to:

- Define the logical systems and main concepts (including mathematical models of computation) used in the proof of the incompleteness theorems
- Reproduce an outline of the proof of Gödel's Incompleteness Theorems and related results
- Explain the philosophical implications of Gödel's Theorems and related results
- Discuss the Church-Turing thesis and the undecidability phenomenon

Course Policies

The course is subject to the various policies found here:

http://www.ugst.umd.edu/courserelatedpolicies.html

In addition, there is the further e-free policy: please minimize the use of laptops, phones, or other mobile devices during class.