Social Choice Theory and Machine Learning
Lecture 1

Eric Pacuit, University of Maryland

August 5, 2024

https://pacuit.org/esslli2024/social-choice-machine-learning/

Quick Survey

- Who considers themselves primarily a computer scientist?

Quick Survey

- Who considers themselves primarily a computer scientist?
- Who considers themselves primarily a logician?

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- What areas of research did I miss?

Quick Survey

- Who considers themselves primarily a computer scientist?
- Who considers themselves primarily a logician?
- Who considers themselves primarily a philosopher?
- Who considers themselves primarily a linguist?
- What areas of research did I miss?
- Who is familiar with social choice theory?
  - Arrow's Theorem?
  - May's Theorem?
  - Condorcet consistent voting methods?

Preferences

Stuart Russell (2019) proposes three principles “to guide AI researchers and developers in thinking about how to create beneficial AI systems” (p. 172):

1. The machine’s only objective is to maximize the realization of human preferences.
2. The machine is initially uncertain about what those preferences are.
3. The ultimate source of information about human preferences is human behavior.

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social choice theory addresses what it might meant to "maximize the realization of human preferences"?

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Social Choice for AI Ethics and Safety

Social Choice for AI Ethics and Safety
SC4AI23
Berkeley

Social Choice for AI Ethics and Safety
SC4AI'24
EUROPE
ECAI-2024 Santiago de Compostela

https://sites.google.com/view/sc4ai/workshops/sc4ai24e

organized by

Vince Conitzer

Jobst Heitzig

Wes Holliday

Conference on AI, Ethics, and Society

AAAI / ACM conference on
ARTIFICIAL INTELLIGENCE,
ETHICS, AND SOCIETY

https://www.aies-conference.com/

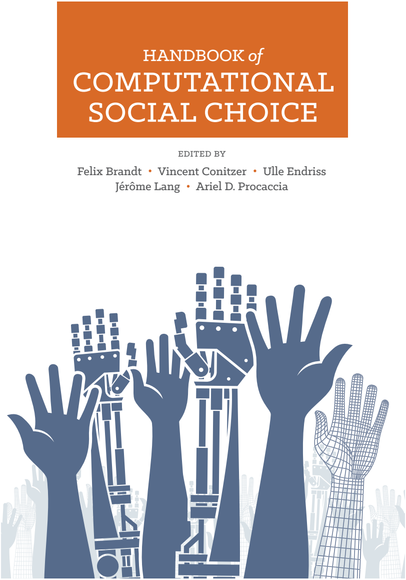
Course Plan

- introduction to mathematical analysis of voting methods, voting paradoxes;
- probabilistic voting methods;
- quantitative analysis of voting methods (e.g., Condorcet efficiency);
- learning voting rules (PAC-learning, MLPs, other approaches);
- using modern deep learning techniques to generate synthetic election data;
- strategic voting, learning to successfully manipulate voting rules based on limited information about how the other voters will vote using neural networks (multi-layer perceptrons);
- RLHF (reinforcement learning with human feedback) and social choice;
- using large-language models to improve group decision-making; and
- liquid democracy (time permitting).

Plan for today (and probably tomorrow)

- A brief introduction to social choice theory
- A survey of voting methods
- Splitting cycles and breaking ties
- (time permitting) Probabilistic voting methods
- Preferential Voting Tools

Background



Let’s review the basic setup of social choice.

Inputs and outputs

Let X be a set of *alternatives*.

voters	rankings
1	$a\ b\ c\ d$
2	$b\ a\ d\ c$
3	$b\ d\ a\ c$
4	$d\ c\ a\ b$

collective decision procedure $\longrightarrow$

utility function on X

lottery on X

ranking of X

winner from X

subset of X (tied winners)

subset of X (multiple winners)

Inputs and outputs

Let X be a set of *alternatives*.

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1	$(a\ b)\ c\ d$
2	$b\ (a\ d\ c)$
3	$b\ d\ a$
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Inputs and outputs

Let X be a set of *alternatives*.

voters	choice fn's
1	$C(\{a,b\}) = \{a\}$ $C(\{a,c\}) = \{a\}$ $\vdots$
2	$C(\{b,a\}) = \{b\}$ $C(\{b,d\}) = \{b\}$ $\vdots$
3	$C(\{b,d\}) = \{b\}$ $C(\{b,a\}) = \{b\}$ $\vdots$
4	$C(\{d,c\}) = \{d\}$ $C(\{d,a\}) = \{d\}$ $\vdots$

collective decision procedure →

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Inputs and outputs

Let X be a set of *alternatives*.

voters	approval sets
1	$\{a, b\}$
2	$\{b, a, d\}$
3	$\{b\}$
4	$\{d, c, a, b\}$

collective decision procedure

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Inputs and outputs

Let X be a set of *alternatives*.

voters	$\mathbb{R}$-valued fn's
1	$\begin{matrix} a & b & c & d \\ .9 & .4 & .2 & .1 \end{matrix}$
2	$\begin{matrix} b & a & d & c \\ .8 & .6 & .4 & .2 \end{matrix}$
3	$\begin{matrix} b & d & a & c \\ .7 & .5 & .3 & .1 \end{matrix}$
4	$\begin{matrix} d & c & a & b \\ .9 & .7 & .5 & .3 \end{matrix}$

collective decision procedure

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collective decision procedure

evaluated by:
axioms satisfied

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evaluated by:

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collective decision procedure

evaluated by:

axioms satisfied
manipulability
complexity
social utility efficiency

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The Voting Problem

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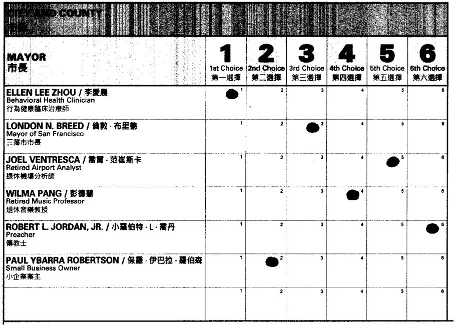
- each voter selects a *ballot* that expresses their preference about the alternatives;
- how should we pick an alternative from X based on the submitted ballots?
- allowing for a tie, we're actually picking a subset of X, and some further (e.g., random) mechanism will choose a final alternative from the subset.

Types of Ballots

Rankings



MAYOR 市長	1 1st Choice 第一選擇	2 2nd Choice 第二選擇	3 3rd Choice 第三選擇	4 4th Choice 第四選擇	5 5th Choice 第五選擇	6 6th Choice 第六選擇
ELLEN LEE ZHOU / 李愛晨 Behavioral Health Clinician 行為健康臨床治療師	1	2	3	4	5	6
LONDON N. BREED / 倫敦 · 布里德 Mayor of San Francisco 三藩市市長	1	2	3	4	5	6
JOEL VENTRESCA / 喬爾 · 范崔斯卡 Retired Airport Analyst 退休機場分析師	1	2	3	4	5	6
WILMA PANG / 彭德慧 Retired Music Professor 退休音樂教授	1	2	3	4	5	6
ROBERT L. JORDAN, JR. / 小羅伯特 · L · 喬丹 Preacher 傳教士	1	2	3	4	5	6
PAUL YBARRA ROBERTSON / 保羅 · 伊巴拉 · 羅伯森 Small Business Owner 小企業業主	1	2	3	4	5	6
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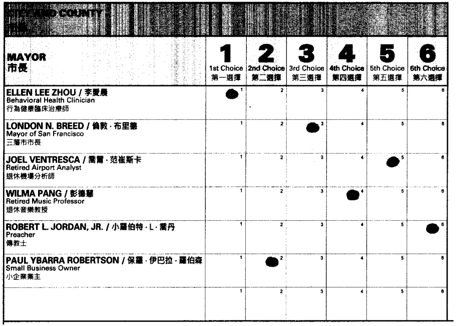

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Choose one

- ☒ Candidate A
- ☐ Candidate B

Choose one

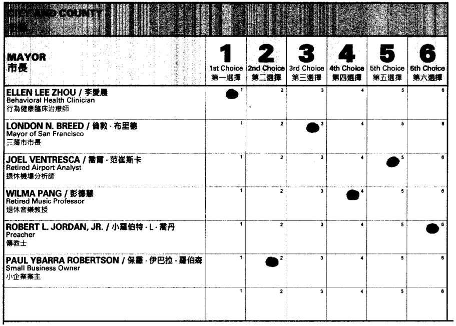
- ☐ Candidate A
- ☒ Candidate C

Choose one

- ☒ Candidate B
- ☐ Candidate C

Types of Ballots

Rankings



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Choose one

- [x] Candidate *A*
- [ ] Candidate *B*

Choose one

- [ ] Candidate *A*
- [x] Candidate *C*

Choose one

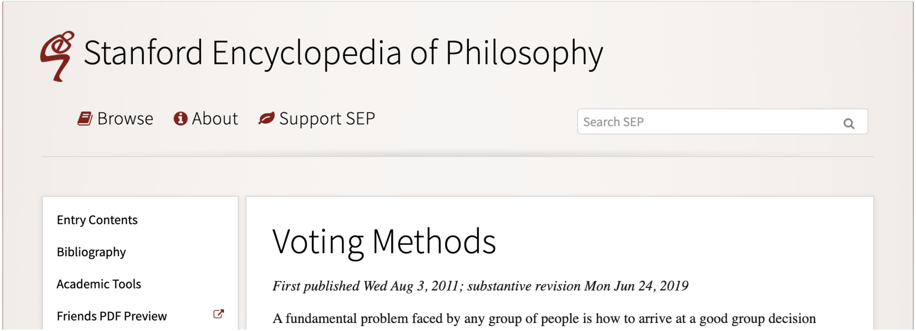
- [x] Candidate *B*
- [ ] Candidate *C*

Grades/Scores

Governor Candidates		Score *each* candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	**0** 1 2 3 4 5 6 7 8 9
2: Candidate B	→	0 1 2 3 4 5 6 7 8 **9**
3: Candidate C	→	0 1 2 3 4 5 6 **7** 8 9

Voting Methods

Many rules have been proposed to choose the winners. See the entry https://plato.stanford.edu/entries/voting-methods/ for an overview.



Rankings

Let X be a set of candidates and V a set of voters.

A **strict linear order** P on X is a relation $P \subseteq X \times X$ satisfying the following conditions for all $x, y, z \in X$:

asymmetry: if $x \, P \, y$ then *not* $y \, P \, x$;

transitivity: if $x \, P \, y$ and $y \, P \, z$, then $x \, P \, z$;

weak completeness: if $x \neq y$, then $x \, P \, y$ or $y \, P \, x$.

Let $\mathcal{L}(X)$ be the set of all strict linear orders on X.

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Let $\mathcal{L}(X)$ be the set of all strict linear orders on X.

We also consider **strict weak orders** on X (denoted $\mathcal{O}(X)$), where voters can submit ties), and may allow voters to submit *truncated* preferences (only rank some of the candidates).

Variable candidate/voter profiles

Fix infinite sets $\mathcal{V}$ and $\mathcal{X}$ of *voters* and *candidates*, respectively.

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A **profile** (of linear orders) is a function $\mathbf{P} : V(\mathbf{P}) \to \mathcal{L}(X(\mathbf{P}))$ for some nonempty finite $V(\mathbf{P}) \subseteq \mathcal{V}$ and nonempty finite $X(\mathbf{P}) \subseteq \mathcal{X}$.

We call $V(\mathbf{P})$ and $X(\mathbf{P})$ the sets of *voters in* $\mathbf{P}$ and *candidates in* $\mathbf{P}$, respectively.

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We call $\mathbf{P}(i)$ voter i's *ranking*, and we write '$x\mathbf{P}_i y$' for $(x, y) \in \mathbf{P}(i)$. As usual, we take $x\mathbf{P}_i y$ to mean that voter i strictly prefers candidate x to candidate y.

Anonymous profiles

40	35	25
t	r	k
k	k	r
r	t	t

The voting problem

40	35	25
t	r	k
k	k	r
r	t	t

Who should win?

The voting problem

40	35	25
t	r	k
k	k	r
r	t	t

Who should win?

- t has the most first place votes (40)

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40	35	25
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Who should win?

- t has the most first place votes (40), but also the most last place votes (40).

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Who should win?

- t has the most first place votes (40), but also the most last place votes (40).
- r beats t if k is dropped from the election (60 to 40).

The voting problem

40	35	25
t	r	k
k	k	r
r	t	t

Who should win?

- t has the most first place votes (40), but also the most last place votes (40).
- r beats t if k is dropped from the election (60 to 40).
- k beats both t (60 to 40) and r (65 to 35) head-to-head.

Scoring Rules and Iterative Methods

2	2	2	1	1	1
a	d	c	b	d	c
b	a	b	d	b	a
d	b	a	a	c	b
c	c	d	c	a	d

Scoring Rules and Iterative Methods

2	2	2	1	1	1
a	d	c	b	d	c
b	a	b	d	b	a
d	b	a	a	c	b
c	c	d	c	a	d

Plurality: c, d

Borda: b

Scoring Rules: Assign *scores* to candidates based on the rankings of the voters. The alternatives with the greatest score are the winners.

Scoring Rules and Iterative Methods

2	2	2	1	1	1
a	d	c	b	d	c
b	a	b	d	b	a
d	b	a	a	c	b
c	c	d	c	a	d

Plurality: c, d

Borda: b

Instant Runoff Voting: d

Coombs: a

Scoring Rules: Assign *scores* to candidates based on the rankings of the voters. The alternatives with the greatest score are the winners.

Iterative Methods: Iteratively remove "poorly performing" candidates until there is a candidate with a majority of first-place votes.

Positional scoring rules

A *scoring vector* is a vector $\langle s_1, \ldots, s_n \rangle$ of numbers such that for each $m \in \{1, \ldots, n-1\}$, $s_m \geq s_{m+1}$.

Given a profile $\mathbf{P}$ with $|X(\mathbf{P})| = n$, $x \in X(\mathbf{P})$, a scoring vector $\vec{s}$ of length n, and $i \in V(\mathbf{P})$, define $\mathit{score}_{\vec{s}}(x, \mathbf{P}_i) = s_r$ where $r = \mathit{Rank}(x, \mathbf{P}_i)$.

Let $\mathit{score}_{\vec{s}}(x, \mathbf{P}) = \sum_{i \in V(\mathbf{P})} \mathit{score}_{\vec{s}}(x, \mathbf{P}_i)$. A voting method F is a **positional scoring rule** if there is a map $\mathcal{S}$ assigning to each natural number n a scoring vector of length n such that for any profile $\mathbf{P}$ with $|X(\mathbf{P})| = n$,

$$F(\mathbf{P}) = \mathrm{argmax}_{x \in X(\mathbf{P})} \mathit{score}_{\mathcal{S}(n)}(x, \mathbf{P}).$$

Examples

Borda:	$\mathcal{S}(n) = \langle n-1, n-2, \ldots, 1, 0\rangle$
Plurality:	$\mathcal{S}(n) = \langle 1, 0, \ldots, 0\rangle$
Anti-Plurality:	$\mathcal{S}(n) = \langle 1, 1, \ldots, 1, 0\rangle$

1	3	2	4
a	b	b	c
c	a	c	a
b	c	a	b

Borda winner	c
Plurality winner	b
Anti-Plurality winner	a

Plurality vs. Borda

1	1
a	c
b	b
c	a

Plurality winners: a, c

Borda winners: a, b, c

Iterative Method: Instant Runoff Voting

- If some alternative is ranked first by an absolute majority of voters, then it is declared the winner.
- Otherwise, the alternative ranked first be the fewest voters (the plurality loser) is eliminated.
- Votes for eliminated alternatives get transferred: delete the removed alternatives from the ballots and "shift" the rankings (e.g., if 1st place alternative is removed, then your 2nd place alternative becomes 1st).

Also known as Ranked-Choice, STV, Hare

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How should you deal with ties? (e.g., multiple alternatives are plurality losers)

Tiebreaking I

- Non-neutral tiebreaking: Fix a linear ordering of the candidates
- Remove all: Remove all candidates tied for the smallest plurality score
- Parallel universe tiebreaking: A candidate a wins if a wins according to some linear ordering of the candidates

S. Obraztsova, E. Elkind and N. Hazon. *Ties Matter: Complexity of Voting Manipulation Revisited*. Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence.

J. Wang, S. Sikdar, T. Shepherd, Z. Zhao, C. Jiang and L. Xia. *Practical Algorithms for Multi-Stage Voting Rules with Parallel Universes Tiebreaking*. Proceedings of AAAI, 2019.

Tiebreaking I

Remove all: Remove all candidates tied for the smallest plurality score

Parallel universe tiebreaking: A candidate a wins if a wins according to some linear ordering of the candidates

1	3	2	1	1
c	c	b	a	a
a	b	a	c	b
b	a	c	b	c

Instant Runoff: $\{c\}$ Instant Runoff PUT: $\{a, c\}$

Iterative Methods

Variants:

- Plurality with runoff: remove all candidates except top two plurality score;
- Coombs: remove candidates with most last place votes;
- Baldwin: remove candidate with smallest Borda score;
- Strict Nanson: remove candidates with below average Borda score

Example

1	1	1	1	1
c	b	a	b	d
a	d	b	c	a
d	a	c	d	b
b	c	d	a	c

Instant Runoff Voting	$\{b\}$
Coombs	$\{d\}$
Baldwin	$\{a, b, d\}$
Strict Nanson	$\{a\}$

Margin Graph

2	2	2	1	1	1
a	d	c	b	d	c
b	a	b	d	b	a
d	b	a	a	c	b
c	c	d	c	a	d

The **margin** of x over y is the number of voters that rank x strictly above y minus the number of voters that rank y strictly above x.

Margin Graph

2	2	2	1	1	1
a	d	c	b	d	c
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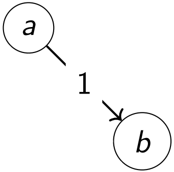

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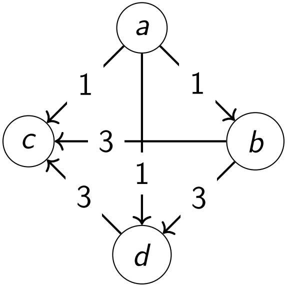
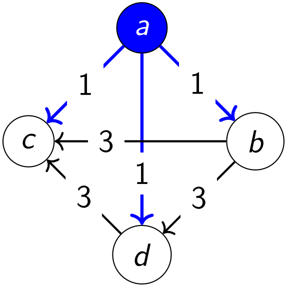
d	b	a	a	c	b
c	c	d	c	a	d



The **margin** of x over y is the number of voters that rank x strictly above y minus the number of voters that rank y strictly above x.

Margin Graph

2	2	2	1	1	1
a	d	c	b	d	c
b	a	b	d	b	a
d	b	a	a	c	b
c	c	d	c	a	d



The **Condorcet winner** is an alternative that is majority preferred to each of the other alternatives.

Margin

Let $\mathbf{P}$ be a profile and $a, b \in X(\mathbf{P})$. Then the **margin of** a **over** b is:

$$Margin_{\mathbf{P}}(a, b) = |\{i \in V(\mathbf{P}) \mid a\mathbf{P}_i b\}| - |\{i \in V(\mathbf{P}) \mid b\mathbf{P}_i a\}|.$$

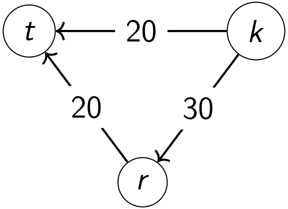
We say that a is **majority preferred** to b in $\mathbf{P}$ when $Margin_{\mathbf{P}}(a, b) > 0$.

Margin Graph

The **margin graph of P**, $\mathcal{M}(\mathbf{P})$, is the weighted directed graph whose set of nodes is $X(\mathbf{P})$ with an edge from a to b weighted by $Margin(a, b)$ when $Margin(a, b) > 0$. We write

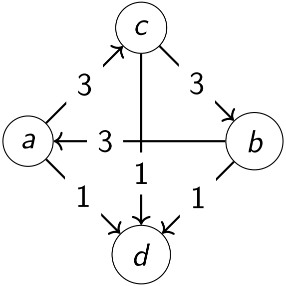
$$a \xrightarrow{\alpha}_{\mathbf{P}} b \text{ if } \alpha = Margin_{\mathbf{P}}(a, b) > 0.$$

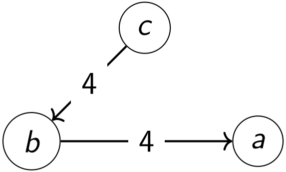
40	35	25
t	r	k
k	k	r
r	t	t



Margin Graph

A **margin graph** is a weighted directed graph $\mathcal{M}$ where all the weights have the same parity.

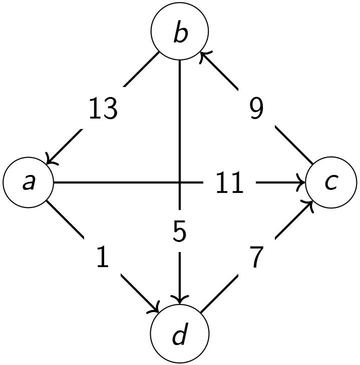




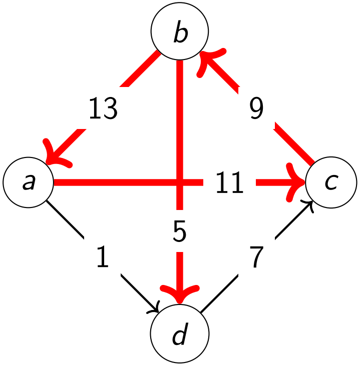
Theorem (Debord, 1987)

If $\mathcal{M}$ is a margin graph with all the weights having the same parity and if there is no edge between any two candidates, then all the weights are even, then there is a profile $\mathbf{P}$ of linear orders such that $\mathcal{M}$ is the margin graph of $\mathbf{P}$.

Margin-Based Methods

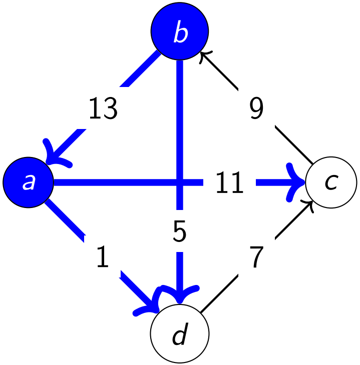


Margin-Based Methods



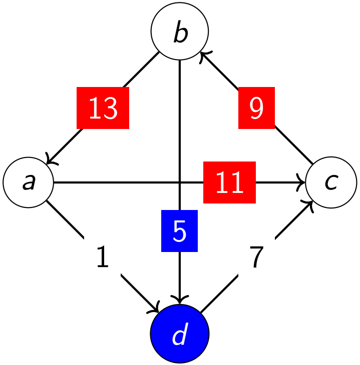
Every candidate loses to at least one other candidate

Margin-Based Methods



Copeland: $\{a, b\}$

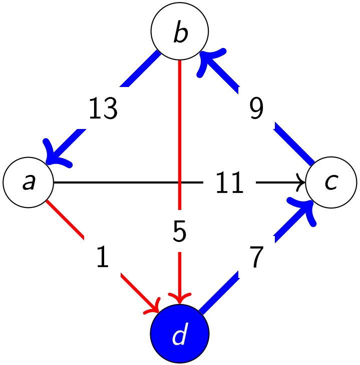
Margin-Based Methods



Copeland: $\{a, b\}$

Minimax: $\{d\}$

Margin-Based Methods

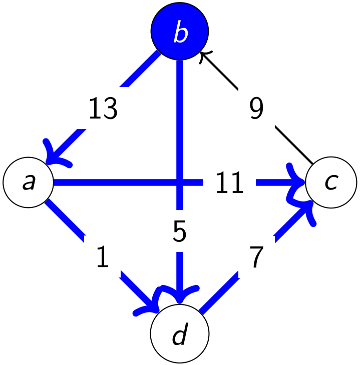


Copeland: $\{a, b\}$

Minimax: $\{d\}$

Beat Path: $\{d\}$

Margin-Based Methods

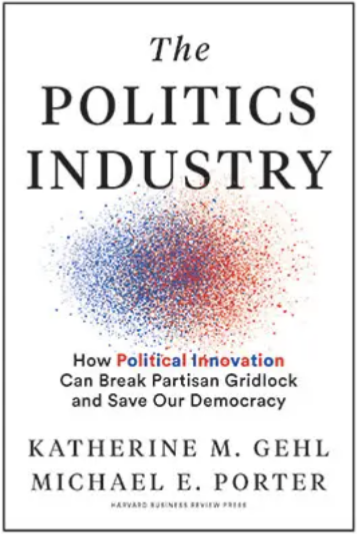


Copeland: $\{a, b\}$

Minimax: $\{d\}$
Beat Path: $\{d\}$
Ranked Pairs: $\{b\}$

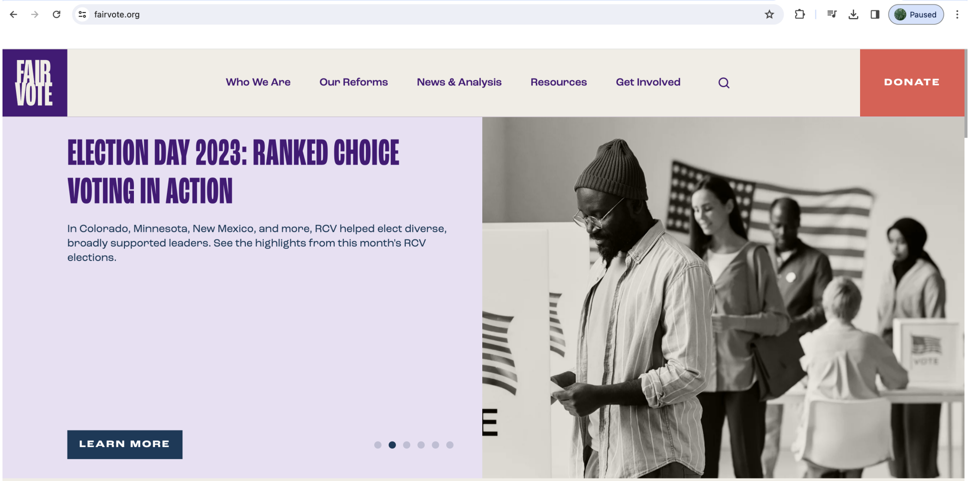
Since different voting methods may select different alternatives for the same input, we need a way to discriminate between different voting methods.

Choosing how to choose



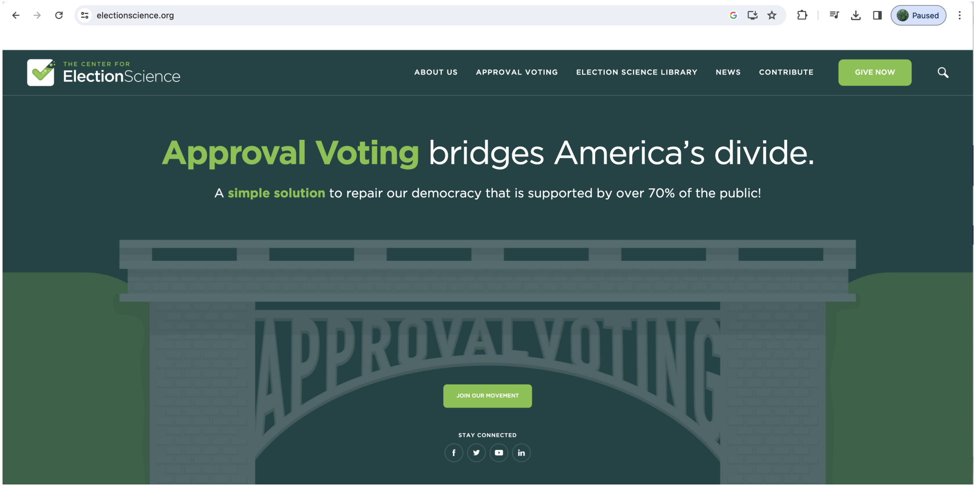
Choosing how to choose

https://FairVote.org



Choosing how to choose

https://electionscience.org/



2022 Alaska Special General Election

Instant Runoff Voting (aka Ranked Choice Voting) winner: Peltola.

2022 Alaska Special General Election

Instant Runoff Voting (aka Ranked Choice Voting) winner: Peltola.

- Three main candidates: Begich, Palin, and Peltola
- Begich is removed in the first round
- Palin loses to Peltola

Round 1

Candidate	Votes	Percentage
Begich, Nick	53,810	28.53%
Palin, Sarah	58,973	31.27%
Peltola, Mary S.	75,799	40.19%
Continuing Ballots Total	**188,582**	
Blanks	3,412	
Exhausted	0	
Overvotes	295	
Remainder Points	0	
Non Transferable Total	**3,707**	

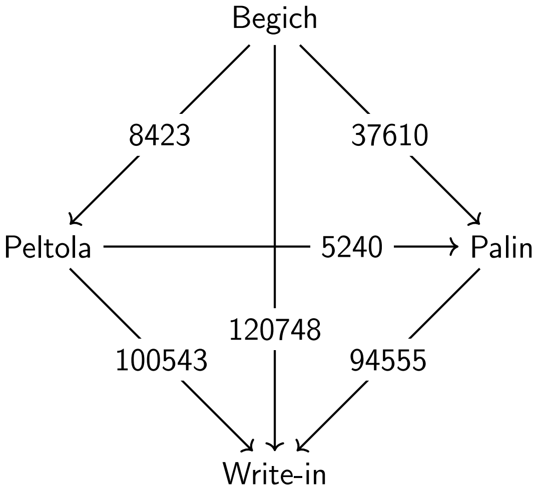
Round 2

Candidate	Votes	Percentage
Begich, Nick	0	0.00%
Palin, Sarah	86,026	48.52%
Peltola, Mary S.	91,266	51.48%
Continuing Ballots Total	**177,292**	
Blanks	3,412	
Exhausted	11,243	
Overvotes	342	
Remainder Points	0	
Non Transferable Total	**14,997**	

2022 Alaska Special General Election

Instant Runoff Voting (IRV) winner: Peltola.

- The write-ins are initially removed
- Begich is removed in the first round
- Palin loses to Peltola



https://github.com/voting-tools/election-analysis/blob/main/alaska_2022.ipynb

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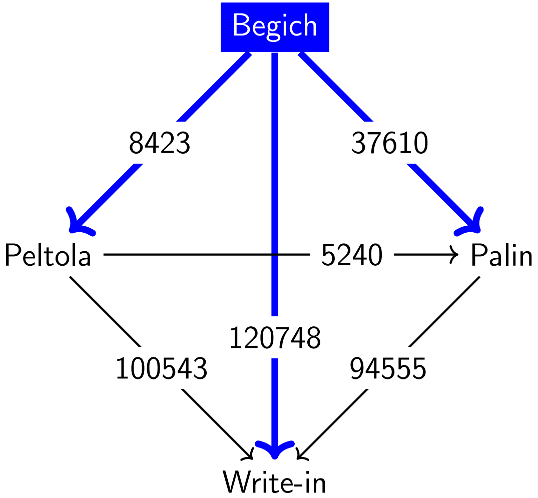

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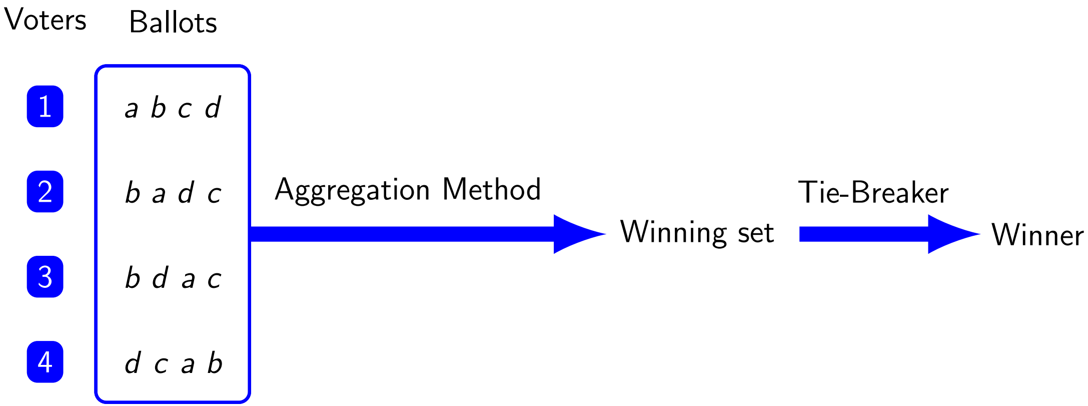
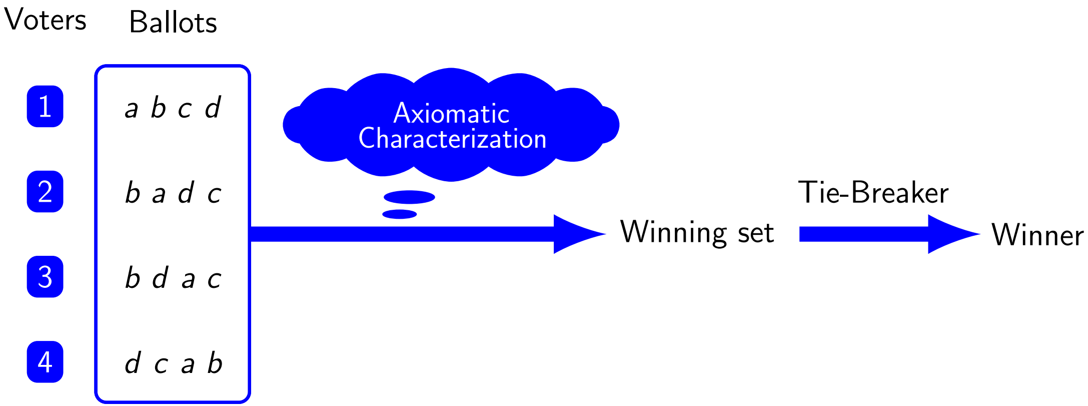


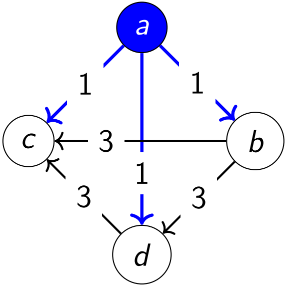
Voters
Ballots
1
2
3
4
a b c d
b a d c
b d a c
d c a b
Aggregation Method
Winning set
Tie-Breaker
Winner



The traditional approach is to identify appealing principles (called axioms) and check which voting methods satisfy these principles.

Some Axioms

Condorcet consistency: If a Condorcet winner exists, then it should be the unique winner.

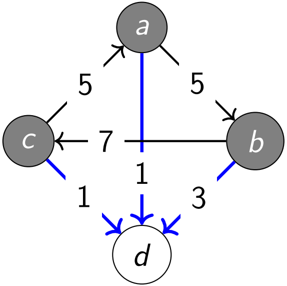


- ✗ Plurality
- ✗ Borda
- ✗ Instant Runoff
- ✗ Coombs

- ✓ Copeland
- ✓ Beat Path
- ✓ Ranked Pairs
- ✓ Minimax

Some Axioms

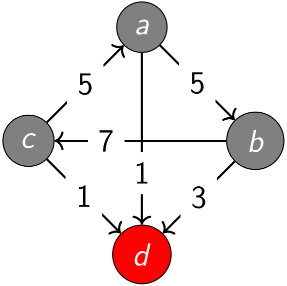
Smith criterion: Always select an alternative from the the smallest set of alternatives such that every alternative in that set is majority preferred to every alternative outside of that set (this set of alternatives is called the Smith set).



✗ Plurality	✓ Copeland
✗ Borda	✓ Beat Path
✗ Instant Runoff	✓ Ranked Pairs
✗ Coombs	✗ Minimax

Some Axioms

Independence of Smith-Dominated Alternatives: The set of winners does not change after removing alternatives that are not in the Smith set.



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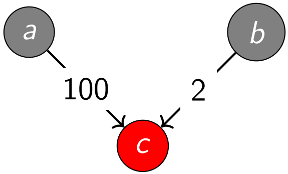

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Characterizing Voting Methods

Going beyond checking which axioms are satisfied, one can prove that a voting method is the unique one satisfying a set of axioms.

Characterizing Voting Methods

Going beyond checking which axioms are satisfied, one can prove that a voting method is the unique one satisfying a set of axioms.

- Majority Rule for 2 candidates (May 1952; Asan and Sanver 2002)
- Plurality Rule (Richelson 1978; Ching 1996; Sekiguchi 2012)
- Borda (Young 1974; Nitzan and Rubinstein 1981; Maskin 2023)
- Instant Runoff Voting (Freeman, Brill, and Conitzer 2014)
- Any positional scoring rule (Young 1975)
- Copeland (Henriet 1985)
- Minimax for 3 candidates (Holliday and Pacuit, under submission, 2024)
- Split Cycle (Ding, Holliday, and Pacuit, forthcoming, 2024)