Aggregating Judgements: Logical and Probabilistic Approaches

Lecture 2

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Plan

- ✓ **Monday** Representing judgements; Introduction to judgement aggregation; Aggregation paradoxes I
- **Tuesday** Aggregation paradoxes II, Axiomatic characterizations of aggregation methods I
- **Wednesday** Axiomatic characterizations of aggregation methods II, Distance-based characterizations
- **Thursday** Opinion pooling; Merging of probabilistic opinions (Blackwell-Dubins Theorem); Aumann's agreeing to disagree theorem and related results
- **Friday** Belief polarization; Diversity trumps ability theorem (The Hong-Page Theorem)

Yesterday

- Aggregating judgements: single event, multiple issues, logically connected issues, probabilistic opinions, imprecise probabilities, causal models, ...
- May's Theorem: axiomatic characterization of majority rule
- Condorcet Jury Theorem: epistemic analysis of majority rule
- Aggregation paradoxes: multiple election paradox

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"Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is." (pg. 234).

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

Anscombe's Paradox

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. Analysis, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	Yes
Voter 5	Yes	No	Yes

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	Yes
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	☺	☺	☺
Voter 5	☺	☺	☺
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	☺	Yes	No
Voter 2	No	☺	No
Voter 3	No	Yes	☺
Voter 4	Yes	No	Yes
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues
Voters 1, 2 & 3 do not support the outcome on a majority of issues

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	Yes
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues
Voters 1, 2 & 3 do not support the outcome on a majority of issues

Avoiding Anscombe's Paradox

The 3/4-Rule: For each proposal, if the set of voters that agree with the outcome of voting on that proposal is at least **three-fourths** of the number of voters (whatever the decision method employed to determine the outcome), then the set of voters who disagree with a majority of the outcomes cannot comprise a majority.

C. Wagner. *Anscombe's paradox and the rule of three-fourths*. Theory and Decision, 15, pgs. 303 - 308, 1983.

G. Laffond and J. Jainé. *Unanimity and the Anscombe's Paradox*. Top, 21, pgs. 590 - 611, 2013.

The Doctrinal Paradox/Discursive Dilemma

Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

P. Mongin. *The doctrinal paradox, the discursive dilemma, and logical aggregation theory*. Theory and Decision, 73(3), pp 315 - 355, 2012.

C. List and P. Pettit. *Aggregating sets of judgments: An impossibility result*. Economics and Philosophy 18, pp. 89 - 110, 2002.

Suppose that three experts *independently* form opinions about three propositions. For instance,

1. c: “Carbon dioxide emissions are above the threshold x.”
2. $c \to g$: “If carbon dioxide emissions are above the threshold x, then there will be global warming.”
3. g: “There will be global warming.”

	c	$c \to g$	g
Expert 1			
Expert 2			
Expert 3			

	c	$c \to g$	g
Expert 1	True	True	
Expert 2			
Expert 3			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2			
Expert 3			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

	c	$c \rightarrow g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True		

	c	$c \rightarrow g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	

	c	$c \rightarrow g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	False

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	False

Should we hire (h) candidate C?

Is C good at research (r)? Is C good at teaching (t)?

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Is C good at research (r)? Is C good at teaching (t)?

	r	t		h
Voter 1				
Voter 2				
Voter 3				
Group				

Should we hire (h) candidate C?
Is C good at research (r)? Is C good at teaching (t)?
We should hire (h) if and only if $r \wedge t$.

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1			Yes	
Voter 2			Yes	
Voter 3			Yes	
Group			Yes	

Should we hire (h) candidate C?
Is C good at research (r)? Is C good at teaching (t)?
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Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group			Yes	

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	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group	Yes	Yes	Yes	No

Suppose that there are five experts $\{1, 2, 3, 4, 5\}$ that are asked about five atomic sentences $\{p_1, p_2, p_3, p_4, p_5\}$ and the disjunction $p_1 \vee p_2 \vee p_3 \vee p_4 \vee p_5$.

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Suppose that each expert i, believes p_i, disbelieves each of the other atomic propositions and believes the disjunction.

There is unanimous support for the disjunction $p_1 \vee p_2 \vee p_3 \vee p_4 \vee p_5$, but 0.8 support *against* each disjunct (i.e., for the negation of each disjunct).

F. Cariani. *Local Supermajorities*. Erkenntnis, 81(2), pp. 391 - 406, 2016.

The Conjunction "Problem"

"In most civil cases, the plaintiff's contention consists of several component elements. So the multiplication law for the mathematical probability of a conjunction entails that, if the contention as a whole is to be established on the balance of mathematical probability, there must either be very few separate components in the case or most of them must be established at a very high level of probability. Since this constraint on the complexity of civil cases is unknown to the law, the mathematicist analysis is in grave difficulties here. "

L. J. Cohen. *The Difficulty about Conjunction*. in *The Probable and The Provable*, 1977.

The Conjunction “Problem”

In a three element negligence case (breach of duty, causation, damages), a plaintiff who proves each element to a 0.6 probability, will have proven her overall claim to a very low probability of 0.216. Either the plaintiff wins the verdict based on this low probability (if the jury focuses on elements) or the plaintiff loses despite having met the condition of proving each element to the stated threshold.

A. P. Dawid. *The Difficulty About Conjunction*. Journal of the Royal Statistical Society. Series D (The Statistician), 36(2/3), pp. 91- 97, 1987.

D. S. Schwartz and E. Sober. *The Conjunction Problem and the Logic of Jury Findings*. William & Mary Law Review 619 (2017).

The Corroboration Paradox

The corroboration paradox is said to occur whenever a sequence of evidentiary propositions, each positively relevant to some hypothesis, fail to be mutually corroborating with respect to that hypothesis.

Theorem If $E_1, \ldots E_n$ are qualitatively independent, then for every family $\{c_I\}_{I \subseteq [n]}$ of real numbers belonging to the open interval $(0, 1)$, there exists a probability measure p defined on the algebra $\mathcal{A}$ generated by $E_1, \ldots, E_n$ and H such that

$$p(H \mid E_I) = c_i \text{ for every } I \subseteq [n]$$

C. Wagner. *The corroboration paradox*. Synthese, 190(8), pp. 1455 - 1469, 2013.

Philosophy department: Should we hire a logician, epistemologist or a metaphysician?

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	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

Philosophy department: Should we hire a logician, epistemologist or a metaphysician?

	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

University: You can't hire three people.

U. Endriss. *Judgment Aggregation with Rationality and Feasibility Constraints*. In Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2018).

Philosophy department: Should we hire a logician, epistemologist or a metaphysician? (**Rationality constraint**: $e \vee l \vee m$)

	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

University: You can't hire three people. (**Feasibility constraint**: $\neg(e \wedge l \wedge m)$)

U. Endriss. *Judgment Aggregation with Rationality and Feasibility Constraints*. In Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2018).

Electing Diverse Committees

Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

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Choose a committee that consists of members from different parts of the university *and* is diverse.

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Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

Electing Diverse Committees

Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

Winners: Bob, David, Fred

Electing Diverse Committees

T. Ratliff. *Selecting committees*. Public Choice, 126, pp. 242 - 255, 2006.

T. Ratliff. *Some startling inconsistencies when electing committees*. Social Choice and Welfare, 21(3), pp. 433- 454, 2003.

T. Ratliff and D. Saari. *Complexities of electing diverse committees*. Social Choice and Welfare, 43(1), pp. 55 - 71, 2014.

Taking stock

- Aggregating judgements: single event, multiple issues, logically connected issues, probabilistic opinions, imprecise probabilities, causal models, ...
- May's Theorem: axiomatic characterization of majority rule
- Condorcet Jury Theorem: epistemic analysis of majority rule
- Aggregation paradoxes: multiple election paradox, doctrinal paradox, discursive dilemma, the problem with conjunction, the corroboration paradox

Judgement Aggregation

U. Endriss. *Judgment Aggregation*. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. D. Procaccia, editors, Handbook of Computational Social Choice, Cambridge University Press, 2016.

C. List. *The theory of judgment aggregation: An introductory review*. Synthese 187(1): 179-207, 2012.

D. Grossi and G. Pigozzi. *Judgement Aggregation: A Primer*. Morgan & Claypool Publishers, 2014.

Propositions: Let $\mathcal{L}$ be a propositional language (with the usual Boolean connectives).

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Issues: $I \subseteq \mathcal{L}$

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Agenda: $A = \{p \mid p \in I\} \cup \{\neg p \mid p \in I\}$

Propositions: Let $\mathcal{L}$ be a propositional language (with the usual Boolean connectives).

Issues: $I \subseteq \mathcal{L}$

Agenda: $A = \{p \mid p \in I\} \cup \{\neg p \mid p \in I\}$

Judgement set for i: $J_i \subseteq A$ that is consistent and complete:

- Consistency: Standard notion of consistency for propositional logic.
- Completeness: For all $\varphi \in I$, $\varphi \in J_i$ or $\neg\varphi \in J_i$.

Notation:

- $\mathscr{J} = \{J \mid J \subseteq A$ is consistent and complete $\}$.
- If $J_i \subseteq \mathcal{L}$, we write $J_i(p) = 1$ when $p \in J_i$ and $J_i(p) = 0$ when $p \notin J_i$.
- If $\mathbf{J} = (J_1, \ldots, J_n)$, then let $\mathbf{J}_p = \{i \mid p \in J_i\}$

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- If $\mathbf{J} = (J_1, \ldots, J_n)$, then let $\mathbf{J}_p = \{i \mid p \in J_i\}$

Aggregation function: $F : \mathcal{J}^n \to \wp(A)$

Proposition-wise Majority:
$F_{maj}(\mathbf{J}) = \{p \mid p \in A$ and $|\mathbf{J}_p| > \lceil n/2 \rceil\}$

Unanimity:
$F_u(\mathbf{J}) = \{p \mid p \in A$ and $|\mathbf{J}_p| = n\}$

Threshold:
$F_t(\mathbf{J}) = \{p \mid p \in A$ and $|\mathbf{J}_p|/n > t_p\}$

where $t = (t_p)_{p \in A}$ is a sequence of thresholds, one for each $p \in A$.

Partition A into **premises** and **conclusions**: $A = Prem \cup Conc$ where $Prem \cap Conc = \emptyset$

For $\mathbf{J} = (J_1, \ldots, J_n)$, let $\mathbf{J}^{Prem} = (J_1 \cap Prem, \ldots, J_n \cap Prem)$ and $\mathbf{J}^{Conc} = (J_1 \cap Conc, \ldots, J_n \cap Conc)$

Partition A into **premises** and **conclusions**: $A = Prem \cup Conc$ where $Prem \cap Conc = \emptyset$

For $\mathbf{J} = (J_1, \ldots, J_n)$, let $\mathbf{J}^{Prem} = (J_1 \cap Prem, \ldots, J_n \cap Prem)$ and $\mathbf{J}^{Conc} = (J_1 \cap Conc, \ldots, J_n \cap Conc)$

Premise-based:
$F_{pb}(\mathbf{J}) = F_{maj}(\mathbf{J}^{Prem}) \cup \{p \mid F_{maj}(\mathbf{J}^{Prem}) \models p\}$
where '$\models$' is the usual notion of logical consequence.

Conclusion-based:
$F_{cb}(\mathbf{J}) = F_{maj}(\mathbf{J}^{Conc})$

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0
F_u			

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0
F_u			
$F_{(\frac{2}{3},\frac{2}{3},\frac{2}{3})}$	1	1	0

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0
F_u			
$F_{(\frac{2}{3},\frac{2}{3},\frac{2}{3})}$	1	1	0
$F_{(\frac{2}{3},1,\frac{2}{3})}$	1		0

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0
F_u			
$F_{(\frac{2}{3},\frac{2}{3},\frac{2}{3})}$	1	1	0
$F_{(\frac{2}{3},1,\frac{2}{3})}$	1		0
F_{pb}	1	1	1

	a	$a \to b$	b
v_1	1	1	1
v_2	1	0	0
v_3	0	1	0
F_{maj}	1	1	0
F_u			
$F_{(\frac{2}{3},\frac{2}{3},\frac{2}{3})}$	1	1	0
$F_{(\frac{2}{3},1,\frac{2}{3})}$	1		0
F_{pb}	1	1	1
F_{cb}			0

Input Condition

Universal Domain: The domain of F is the set of all possible profiles of consistent and complete judgement sets.

Output Condition

Collective Rationality: F generates consistent and complete collective judgment sets.

Responsiveness Conditions

Systematicity: For any $p, q \in A$ and all $\mathbf{J} = (J_1, \ldots, J_n)$ and $\mathbf{J}^* = (J_1^*, \ldots, J_n^*)$ in the domain of F,

if [for all $i \in N$, $p \in J_i$ iff $q \in J_i^*$]

then [$p \in F(\mathbf{J})$ iff $q \in F(\mathbf{J}^*)$].

$A = \{p_1, \dots, p_m\}$

$\mathbf{J} = (J_1, \dots, J_n)$

$$
\begin{array}{cccccc}
p_1 & p_2 & \cdots & p_k & \cdots & p_m \\
\hline
J_1(p_1) & J_1(p_2) & \cdots & J_1(p_k) & \cdots & J_1(p_m) \\
J_2(p_1) & J_2(p_2) & \cdots & J_2(p_k) & \cdots & J_2(p_m) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
J_l(p_1) & J_l(p_2) & \cdots & J_l(p_k) & \cdots & J_l(p_m) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
J_n(p_1) & J_n(p_2) & \cdots & J_n(p_k) & \cdots & J_n(p_m) \\
\hline
F(\mathbf{J})(p_1) & F(\mathbf{J})(p_2) & \cdots & F(\mathbf{J})(p_k) & \cdots & F(\mathbf{J})(p_m) \\
\hline
\end{array}
$$

$A = \{p_1, \ldots, p_m\}$

$\mathbf{J} = (J_1, \ldots, J_n)$, $\mathbf{J}^* = (J_1, \ldots, J_n)$

p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1^*(p_1)$	$J_1^*(p_2)$	$\cdots$	$J_1^*(p_k)$	$\cdots$	$J_1^*(p_m)$
$J_2^*(p_1)$	$J_2^*(p_2)$	$\cdots$	$J_2^*(p_k)$	$\cdots$	$J_2^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l^*(p_1)$	$J_l^*(p_2)$	$\cdots$	$J_l^*(p_k)$	$\cdots$	$J_l^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n^*(p_1)$	$J_n^*(p_2)$	$\cdots$	$J_n^*(p_k)$	$\cdots$	$J_n^*(p_m)$
$F(\mathbf{J}^*)(p_1)$	$F(\mathbf{J}^*)(p_2)$	$\cdots$	$F(\mathbf{J}^*)(p_k)$	$\cdots$	$F(\mathbf{J}^*)(p_m)$

$A = \{p_1, \ldots, p_m\}$

$\mathbf{J} = (J_1, \ldots, J_n)$, $\mathbf{J}^* = (J_1, \ldots, J_n)$

p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1(p_1)$	$J_1(p_2)$	$\cdots$	$J_1(p_k)$	$\cdots$	$J_1(p_m)$
$J_2(p_1)$	$J_2(p_2)$	$\cdots$	$J_2(p_k)$	$\cdots$	$J_2(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l(p_1)$	$J_l(p_2)$	$\cdots$	$J_l(p_k)$	$\cdots$	$J_l(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n(p_1)$	$J_n(p_2)$	$\cdots$	$J_n(p_k)$	$\cdots$	$J_n(p_m)$
$F(\mathbf{J})(p_1)$	$F(\mathbf{J})(p_2)$	$\cdots$	$F(\mathbf{J})(p_k)$	$\cdots$	$F(\mathbf{J})(p_m)$

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p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1(p_1)$	1	$\cdots$	$J_1(p_k)$	$\cdots$	$J_1(p_m)$
$J_2(p_1)$	0	$\cdots$	$J_2(p_k)$	$\cdots$	$J_2(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l(p_1)$	1	$\cdots$	$J_l(p_k)$	$\cdots$	$J_l(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n(p_1)$	1	$\cdots$	$J_n(p_k)$	$\cdots$	$J_n(p_m)$
$F(\mathbf{J})(p_1)$	1	$\cdots$	$F(\mathbf{J})(p_k)$	$\cdots$	$F(\mathbf{J})(p_m)$

$A = \{p_1, \ldots, p_m\}$

$\mathbf{J} = (J_1, \ldots, J_n)$, $\mathbf{J}^* = (J_1, \ldots, J_n)$

p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1^*(p_1)$	$J_1^*(p_2)$	$\cdots$	1	$\cdots$	$J_1^*(p_m)$
$J_2^*(p_1)$	$J_2^*(p_2)$	$\cdots$	0	$\cdots$	$J_2^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l^*(p_1)$	$J_l^*(p_2)$	$\cdots$	1	$\cdots$	$J_l^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n^*(p_1)$	$J_n^*(p_2)$	$\cdots$	1	$\cdots$	$J_n^*(p_m)$
$F(\mathbf{J}^*)(p_1)$	$F(\mathbf{J}^*)(p_2)$	$\cdots$	$F(\mathbf{J}^*)(p_k)$	$\cdots$	$F(\mathbf{J}^*)(p_m)$

$A = \{p_1, \ldots, p_m\}$

$\mathbf{J} = (J_1, \ldots, J_n)$, $\mathbf{J}^* = (J_1, \ldots, J_n)$

p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1^*(p_1)$	$J_1^*(p_2)$	$\cdots$	1	$\cdots$	$J_1^*(p_m)$
$J_2^*(p_1)$	$J_2^*(p_2)$	$\cdots$	0	$\cdots$	$J_2^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l^*(p_1)$	$J_l^*(p_2)$	$\cdots$	1	$\cdots$	$J_l^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n^*(p_1)$	$J_n^*(p_2)$	$\cdots$	1	$\cdots$	$J_n^*(p_m)$
$F(\mathbf{J}^*)(p_1)$	$F(\mathbf{J}^*)(p_2)$	$\cdots$	1	$\cdots$	$F(\mathbf{J}^*)(p_m)$

$A = \{p_1, \ldots, p_m\}$

$\mathbf{J} = (J_1, \ldots, J_n)$, $\mathbf{J}^* = (J_1, \ldots, J_n)$

p_1	p_2	$\cdots$	p_k	$\cdots$	p_m
$J_1^*(p_1)$	$J_1^*(p_2)$	$\cdots$	1	$\cdots$	$J_1^*(p_m)$
$J_2^*(p_1)$	$J_2^*(p_2)$	$\cdots$	0	$\cdots$	$J_2^*(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l^*(p_1)$	$J_l^*(p_2)$	$\cdots$	1	$\cdots$	$J_l^*(p_m)$
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$J_1(p_1)$	1	$\cdots$	$J_1(p_k)$	$\cdots$	$J_1(p_m)$
$J_2(p_1)$	0	$\cdots$	$J_2(p_k)$	$\cdots$	$J_2(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_l(p_1)$	1	$\cdots$	$J_l(p_k)$	$\cdots$	$J_l(p_m)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$J_n(p_1)$	1	$\cdots$	$J_n(p_k)$	$\cdots$	$J_n(p_m)$
$F(\mathbf{J})(p_1)$	0	$\cdots$	$F(\mathbf{J})(p_k)$	$\cdots$	$F(\mathbf{J})(p_m)$

Responsiveness Conditions

Systematicity: For any $p, q \in A$ and all $\mathbf{J} = (J_1, \ldots, J_n)$ and $\mathbf{J}^* = (J_1^*, \ldots, J_n^*)$ in the domain of F,

if [for all $i \in N$, $p \in J_i$ iff $q \in J_i^*$]

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Independence: For any $p \in A$ and all $\mathbf{J} = (J_1, \ldots, J_n)$ and $\mathbf{J}^* = (J_1^*, \ldots, J_n^*)$ in the domain of F,

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then [$p \in F(\mathbf{J})$ iff $p \in F(\mathbf{J}^*)$].

Responsiveness Conditions

Anonymity: For all profiles $(J_1, \ldots, J_n)$, $F(J_1, \ldots, J_n) = F(J_{\pi(1)}, \ldots, J_{\pi(n)})$ where π is a permutation of the voters.

Unanimity: For all profiles $(J_1, \ldots, J_n)$ if $p \in J_i$ for each i then $p \in F(J_1, \ldots, J_n)$

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Unanimity: For all profiles $(J_1, \ldots, J_n)$ if $p \in J_i$ for each i then $p \in F(J_1, \ldots, J_n)$

Monotonicity: For any $p \in X$ and all $(J_1, \ldots J_i, \ldots, J_n)$ and $(J_1, \ldots, J_i^*, \ldots, J_n)$ in the domain of F,

if [$p \notin J_i$, $p \in J_i^*$ and $p \in F(J_1, \ldots, J_i, \ldots J_n)$]
then [$p \in F(J_1, \ldots, J_i^*, \ldots J_n)$].

Responsiveness Conditions

Non-dictatorship: There exists no $i \in N$ such that, for any profile $(J_1, \ldots, J_n)$, $F(J_1, \ldots, J_n) = J_i$

Baseline Result

Theorem (List and Pettit, 2001) If $X \subseteq \{a, b, a \wedge b\}$, there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.

Agenda Richness

Whether or not judgment aggregation gives rise to serious impossibility results depends on how the propositions in the agenda are *interconnected*.

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Definition A set $Y \subseteq \mathcal{L}$ is **minimally inconsistent** if it is inconsistent and every proper subset $X \subsetneq Y$ is consistent.

Agenda Richness

Definition An agenda X is **minimally connected** if

1. (non-simple) it has a minimal inconsistent subset $Y \subseteq X$ with $|Y| \geq 3$
2. (*even-number-negatable*) it has a minimal inconsistent subset $Y \subseteq X$ such that

   $$Y - Z \cup \{\neg z \mid z \in Z\} \text{ is consistent}$$

   for some subset $Z \subseteq Y$ of even size.

Impossibility Theorems

Theorem (Dietrich and List, 2007) If (and only if) an agenda is non-simple and even-number negatable, every aggregation rule satisfying universal domain, collective rationality, systematicity and unanimity is a dictatorship (or inverse dictatorship).

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Theorem (Dietrich and List, 2007) If (and only if) an agenda is non-simple and even-number negatable, every aggregation rule satisfying universal domain, collective rationality, systematicity and unanimity is a dictatorship (or inverse dictatorship).

Theorem (Nehring and Puppe, 2002) If (and only if) an agenda is non-simple, every aggregation rule satisfying universal domain, collective rationality, systematicity unanimity, and monotonicity is a dictatorship.

Characterization Result

$p \in X$ conditionally entails $q \in X$, written $p \vdash^* q$ provided there is a subset $Y \subseteq X$ consistent with each of p and $\neg q$ such that $\{p\} \cup Y \vdash q$.

Totally Blocked: X is totally blocked if for any $p, q \in X$ there exists $p_1, \ldots, p_k \in X$ such that

$$p = p_1 \vdash^* p_2 \vdash^* \cdots \vdash^* p_k = q$$

Characterization Result

Theorem (Dietrich and List, 2007, Dokow Holzman 2010) If (and only if) an agenda is totally blocked and even-number negatable, every aggregation rule satisfying universal domain, collective rationality, independence and unanimity is a dictatorship.

Theorem (Nehring and Puppe, 2002, 2010) If (and only if) an agenda is totally blocked, every aggregation rule satisfying universal domain, collective rationality, independence unanimity, and monotonicity is a dictatorship.

Proof Sketch, I

$C \subseteq N$ is **winning for** p if for all profiles $\mathbf{A} = (A_1, \ldots, A_n)$, if $p \in A_i$ for all $i \in C$ and $p \notin A_j$ for all $j \notin C$, then $p \in F(\mathbf{A})$

$\mathcal{C}_p = \{C \mid C \text{ is winning for } p\}$

Proof Sketch, II

1. (The agenda is totally blocked.) $C_p = C_q$ for all p, q. Let $C = C_p$ for some p (hence for all p).

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4. $N \in \mathcal{C}$.
5. For all $C \subseteq N$, either $C \in \mathcal{C}$ or $\overline{C} \in \mathcal{C}$.

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4. $N \in \mathcal{C}$.

5. For all $C \subseteq N$, either $C \in \mathcal{C}$ or $\overline{C} \in \mathcal{C}$.

6. There is an $i \in N$ such that $\{i\} \in \mathcal{C}$.