Social Choice Theory for Logicians
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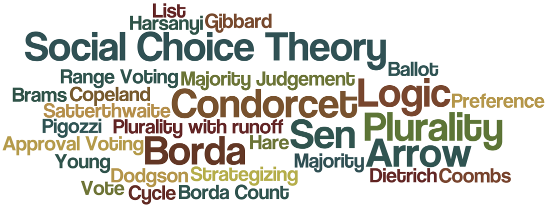

Plan

- Introduction, Background, Voting Theory, May's Theorem, Arrow's Theorem
- Social Choice Theory: Variants of Arrow's Theorem, Weakening Arrow's Conditions (Domain Conditions), Harsanyi's Theorem, Characterizing Voting Methods
- Strategizing (Gibbard-Satterthwaite Theorem) and Iterative Voting/ Introduction to Judgement Aggregation
- Aggregating Judgements (linear pooling, wisdom of the crowds, prediction markets), Probabilistic Social Choice.
- Logics for Social Choice Theory (Preference Logic, Modal Logic, Dependence/Independence Logic, First Order Logic)

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Finding a compromise ranking

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- Given an appropriate notion of distance, what is the definition of a compromise ranking? *mean or median*

J. Kemeny. *Mathematics Without Numbers*. Daedalus, 88, pgs. 571 - 591, 1959.

Kemeny Distance

Key idea: The ranking $A \succ B \succ C$ is *closer* to $A \succ C \succ B$ than to $B \succ C \succ A$.

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Why?

	$\{A, B\}$	$\{B, C\}$	$\{A, C\}$
$A \succ B \succ C$	$A \succ B$	$B \succ C$	$A \succ C$
$A \succ C \succ B$	$A \succ B$	$C \succ B$	$A \succ C$
$B \succ C \succ A$	$B \succ A$	$B \succ C$	$C \succ A$

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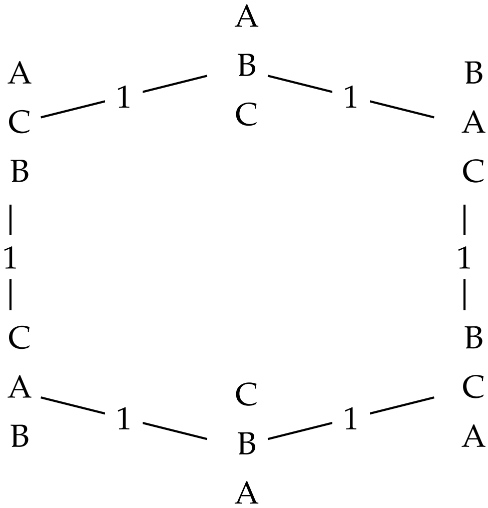
	$\{A, B\}$	$\{B, C\}$	$\{A, C\}$
$A \succ B \succ C$	$A \succ B$	$B \succ C$	$A \succ C$
$A \succ C \succ B$	$A \succ B$	$C \succ B$	$A \succ C$
$B \succ C \succ A$	$B \succ A$	$B \succ C$	$C \succ A$

$K(A \succ B \succ C, A \succ C \succ B) = 1$ because the rankings disagree on one pair of candidates, while $K(A \succ B \succ C, B \succ C \succ A) = 2$ because the rankings disagree on two paris of candidates.

Kemeny Distance



Mean or Median?

Mean or Median?

Let $a_1, \ldots, a_n$ be a set of numbers.

The **mean** is the number x that minimizes the sum of the square of the distance between each data point and x.

The **median** is the number x that minimizes the sum of the distances between each data point and x

Mean or Median?

# voters	21	5	4	11
	A	B	C	C
	B	C	A	B
	C	A	B	A

$K \mid K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
$A\ B\ C$						
$B\ C\ A$						
$C\ A\ B$						
$C\ B\ A$						

$K \mid K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
$A\ B\ C$	$0 \mid 0$	$1 \mid 1$	$1 \mid 1$	$2 \mid 4$	$2 \mid 4$	$3 \mid 9$
$B\ C\ A$	$2 \mid 4$	$3 \mid 9$	$1 \mid 1$	$0 \mid 0$	$2 \mid 4$	$1 \mid 1$
$C\ A\ B$	$2 \mid 4$	$1 \mid 1$	$3 \mid 9$	$2 \mid 4$	$0 \mid 0$	$1 \mid 1$
$C\ B\ A$	$3 \mid 9$	$2 \mid 4$	$2 \mid 4$	$1 \mid 1$	$1 \mid 1$	$0 \mid 0$

$n{*}K \mid n{*}K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
(21) $A\ B\ C$	0 | 0	21 | 21	21 | 21	42 | 84	42 | 84	63 | 189
$B\ C\ A$	2 | 4	3 | 9	1 | 1	0 | 0	2 | 4	1 | 1
$C\ A\ B$	2 | 4	1 | 1	3 | 9	2 | 4	0 | 0	1 | 1
$C\ B\ A$	3 | 9	2 | 4	2 | 4	1 | 1	1 | 1	0 | 0

$n{*}K \mid n{*}K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
(21) $A\ B\ C$	0 | 0	21 | 21	21 | 21	42 | 84	42 | 84	63 | 189
(5) $B\ C\ A$	10 | 20	15 | 45	5 | 5	0 | 0	10 | 20	5 | 5
(4) $C\ A\ B$	8 | 16	4 | 4	12 | 36	8 | 16	0 | 0	4 | 4
(11) $C\ B\ A$	33 | 99	22 | 44	22 | 44	11 | 11	11 | 11	0 | 0
Sum	51 | 135	62 | 114	60 | 106	61 | 111	63 | 115	71 | 198

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# voters	21	5	4	11
	A	B	C	C
	B	C	A	B
	C	A	B	A

- Median Ranking: $A \succ B \succ C$ (Minimizes the sum of the Kemeny distances)
- Mean Ranking: $B \succ A \succ C$ (Minimizes the sum of the square of the Kemeny distances)

Manipulating an election outcome

It has long been noted that a voter can achieve a preferred election outcome by misrepresenting his or her actual preferences.

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> *"adopt a principle of voting which makes it a game of skill than a real test of the wishes of the elector."*

and that in his opinion

> *"it would be better for elections to be decided according to the wishes of the majority than of those who happen to be more skilled at the game."*

Manipulating an election outcome

"If we assume society discourages the concentration of power, then at least two methods of manipulation are always available, no matter what method of voting is used: First, those in control of procedures can manipulate the agenda (by, for example, restricting alternatives or by arranging the order in which they are brought up). Second, those not in control can still manipulate by false revelation of values." (p. 137)

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Literature

A. Taylor. *Social Choice and the Mathematics of Manipulation*. Cambridge University Press, 2005.

W. Poundstone. *Gaming the Vote*. Hill and Wang Publishers, 2008.

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

Manipulation: misrepresenting preferences

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Borda Winner: *B*

Manipulation: misrepresenting preferences

# voters	3	3	1
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	B	A	A
	C	C	B

Borda Winner: *A*

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Borda Winner: *B*

Borda: “My procedure is only meant for honest people!”

Notation: Candidates, Voters

- N is a finite set of voters (assume that $N = \{1, 2, 3, \ldots, n\}$)

- X is a (typically finite) set of alternatives: e.g., candidates, restaurants, social states, etc.

Notation: Preferences

- A relation on X is a linear order if it is transitive, irreflexive, and complete (hence, acyclic)
- $L(X)$ is the set of all linear orders over the set X
- $O(X)$ is the set of all reflexive, transitive and complete relations over the set X
- Given $R \in O(X)$, let the strict subrelation be $P_R = \{(x, y) \mid x\ R\ y \text{ and } y \not R\ x\}$ and the indifference subrelation be $I_R = \{(x, y) \mid x\ R\ y \text{ and } y\ R\ x\}$

Notation: Profiles

- A **profile** for the set of voters N is a sequence of (linear) orders over X, denoted $\mathbf{R} = (R_1, \ldots, R_n)$.

- $L(X)^n$ is the set of all **profiles** for n voters (similarly for $O(X)^n$)

- For a profile $\mathbf{R} = (R_1, \ldots, R_n) \in O(X)^n$, let $\mathbf{N_R}(A\ P\ B) = \{i \mid A\ P_i\ B\}$ be the set of voters that rank A above B (similarly for $\mathbf{N_R}(A\ I\ B)$ and $\mathbf{N_R}(B\ P\ A)$)

Group Decision Making Methods

$$F : \mathcal{D} \to \mathcal{R}$$

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$$F : \mathcal{D} \to \mathcal{R}$$

Comments

- $\mathcal{D}$ is the *domain* of the function: the set of possible "election scenarios" (i.e., $\mathcal{D} \subseteq L(X)^n$, $\mathcal{D} \subseteq O(X)^n$, $\mathcal{D} \subseteq U(X)^n$, where $U(X)$ is the set of utility functions on X)
- The group decision is *completely determined* by the voters' opinions: every profile $\mathbf{R} \in \mathcal{D}$ is associated with exactly one "group decision".

Group Decision Making Methods

$$F : \mathcal{D} \to \mathcal{R}$$

Variants

- Social Welfare Functions: $\mathcal{R} = L(X)$ or $\mathcal{R} = O(X)$.
- Social Choice Function: $\mathcal{R} = \wp(X) - \emptyset$, where $\wp(X)$ is the set of all subsets of X.

A social choice function $F : L(X)^n \to (\wp(X) - \emptyset)$ is **manipulable** provided there are two profiles

$$\mathbf{P} = (P_1, \ldots, P_i, \ldots, P_n) \text{ and } \mathbf{P}' = (P'_1, \ldots, P'_i, \ldots, P'_n)$$

and a voter i such that

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Intuition: P_i is voter i's "true preference".

Fact. Borda count is single-winner manipulable.

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1	1	1	1
A	B	D	C
B	D	C	A
C	C	A	B
D	A	B	D

Borda Winner: C

1	1	1	1
B	B	D	C
A	D	C	A
D	C	A	B
C	A	B	D

Borda Winner: B

$F : L(X)^n \to (\wp(X) - \emptyset)$

Resolute: For all profiles $\mathbf{P} \in L(X)^n$, $F(\mathbf{P})$ is a singelton.

Pareto: For all profiles $\mathbf{P} = (P_1, \ldots, P_n)$ and all pairs of alternatives A, B: if $A\ P_i\ B$ for all $i \in N$, then $B \notin F(\mathbf{P})$

Manipulability: A resolute social choice function is manipulable there there exists a voter $i \in N$, a profile $\mathbf{P} = (P_1, \ldots, P_i, \ldots, P_n)$ and linear ordering $Q_i \in L(X)$ such that $R_i \neq Q_i$ and

$$F(P_1, \ldots, Q_i, \ldots, P_n)\ R_i\ F(P_1, \ldots, P_i, \ldots, P_n)$$

A resolute social choice function is **strategy-proof** if it is not manipulable.

The Gibbard-Satterthwaite Theorem

Theorem. Suppose that X has at least three elements. Any resolute social choice function $F : L(X)^n \to X$ that is Pareto and strategy-proof must be a dictatorship.

M. A. Satterthwaite. *Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions*. Journal of Economic Theory, 10(2):187-217, 1975.

A. Gibbard. *Manipulation of voting schemes: A general result*. Econometrica, 41(4):587-601, 1973.

Questions

- Which assumptions of the theorem or “crucial”? (non-resolute social choice functions, weak orders, partial orders, domain restrictions: single-peaked domains, etc.)

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- How "hard" is it to manipulate an election?
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- What's so *bad* about manipulation?
- How "hard" is it to manipulate an election?
- When will a voter misrepresent her preferences? What do the voters need to *know* (believe) about the other voters' preferences?
- What is the effect of polling information on an election outcome?

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What happens if $F(\mathbf{R})$ and $F(\mathbf{R}')$ are not singletons?

Suppose that $F(\mathbf{R}) = Y$ and $F(\mathbf{R}') = Z$ are not singletons

- Z **weakly dominates** Y for i provided

$$\text{for all } z \in Z \text{ and } y \in Y \;\; z \, R_i \, y \;\; \text{and}$$

$$\text{there exists } z \in Z \text{ and } y \in Y \text{ such that } z \, P_i \, y$$

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- Z is preferred by an **optimist** to Y: $\max_i(Z) \, P_i \, \max_i(Y)$

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- Z is preferred by an **optimist** to Y: $\max_i(Z) \, P_i \, \max_i(Y)$

- Z is preferred by a **pessimist** to Y: $\min_i(Z) \, P_i \, \min_i(Y)$

- Z has higher **expected utility** than Y: There exists a utility function representing P_i such that, if $p_Z(\cdot) = \frac{1}{|Z|}$ and $p_Y(\cdot) = \frac{1}{|Y|}$, then

$$\sum_{z \in Z} p_Z(z) \cdot u(z) > \sum_{y \in Y} p_Y(y) \cdot u(y)$$

Fact. Plurality rules is weak dominance manipulable, but is never single-winner manipulable.

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1	2	1
A	C	B
B	A	A
C	B	C

Plurality Winner: C

1	2	1
B	C	B
A	A	A
C	B	C

Plurality Winners: $\{B, C\}$

Fact. Condorcet rule is manipulable by optimists (and also by pessimists), but is never weak dominance manipulable.

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1	1	1
A	B	C
C	C	A
B	A	B

Condorcet Winner: C

1	1	1
A	B	C
B	C	A
C	A	B

Condorcet Winners: $\{A, B, C\}$

Fact. The “near-unanimity rule” is manipulable by pessimists, but is never by optimists.

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1	1	1
A	B	C
B	A	A
C	C	B

Near-Unanimity Winner: $\{A, B, C\}$

1	1	1
B	B	C
A	A	A
C	C	B

Near-Unanimity Winner: $\{B\}$

Fact. The “Pareto rule” is expected-utility manipulable, but never manipulable by optimists or pessimists.

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1	1	1
A	A	C
B	C	B
C	B	A

Pareto Winner: $\{A, B, C\}$

1	1	1
A	A	C
C	C	B
B	B	A

Pareto Winner: $\{A, C\}$

Fact. The "Pareto rule" is expected-utility manipulable, but never manipulable by optimists or pessimists.

1	1	1
A	A	C
B	C	B
C	B	A

Pareto Winner: $\{A, B, C\}$

1	1	1
A	A	C
C	C	B
B	B	A

Pareto Winner: $\{A, C\}$

Let $u_1(A) = 18, u_1(B) = 9$, and $u_1(C) = 6$

$$18 \cdot \frac{1}{3} + 9 \cdot \frac{1}{3} + 6 \cdot \frac{1}{3} = 11 \qquad 18 \cdot \frac{1}{2} + 6 \cdot \frac{1}{2} = 12$$

The Duggan-Schwartz Theorem

$F : O(X)^n \to \wp(X) - \emptyset$

i is a **nominator** if for all profiles $\mathbf{R} = (R_1, \ldots, R_i, \ldots, R_n)$, $Top(R_i) \in F(\mathbf{R})$.

Non-Imposed: For all $A \in X$ there exists a profile $\mathbf{R}$ such that $F(\mathbf{R}) = \{A\}$.

Manipulated by an optimist: there there exists a voter $i \in N$, a profile $\mathbf{R} = (R_1, \ldots, R_i, \ldots, R_n)$ and a ordering $Q_i \in O(X)$ such that $R_i \neq Q_i$ and

$$\text{there exists } A \in F(\mathbf{R}') \text{ such that for all } B \in F(\mathbf{R}), A\ P_i\ B$$

where $\mathbf{R}' = (R_1, \ldots, Q_i, \ldots, R_n)$

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Manipulated by an pessimist: there there exists a voter $i \in N$, a profile $\mathbf{R} = (R_1, \ldots, R_i, \ldots, R_n)$ and a ordering $Q_i \in O(X)$ such that $R_i \neq Q_i$ and

$$\text{for all } A \in F(\mathbf{R}') \text{ there is a } B \in F(\mathbf{R}),\ A\ P_i\ B$$

where $\mathbf{R}' = (R_1, \ldots, Q_i, \ldots, R_n)$

The Duggan-Schwartz Theorem

Theorem. Suppose that X has at least three elements. Any social choice function $F : O(X)^n \rightarrow (\wp(X) - \emptyset)$ that is non-imposed and cannot be manipulated by an optimist or a pessimist has a nominator

J. Duggan and T. Schwartz. *Strategic manipulability without resoluteness or shared beliefs: Gibbard-Satterthwaite generalized*. Social Choice and Welfare, 17, pgs. 85 - 93, 2000.

What so bad about manipulation?

K. Dowding and M. van Hees. *In Praise of Manipulation*. British Journal of Political Science, 38:1, pgs. 1 - 15, 2008.

Sincerity argument: Manipulation is a form of insincerity or dishonesty that is undesirable or even immoral (“Voters have an incentive to “lie”)

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Sincerity argument: Manipulation is a form of insincerity or dishonesty that is undesirable or even immoral ("Voters have an incentive to "lie")

- Riker's argument against populism: given the results of social choice theory, democracy as a representation of the honest views of the society must be abandoned

Transparency argument: If there is the possibility that voters can manipulate the outcome of an election, then the group decision process is can manipulate the outcome of an election, then the group decision process is "opaque".

Sincere Manipulation

"To forgo pursuing our most preferred outcomes for reason of feasibility is not insincerity"

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(e.g., Nader/Gore voters in the 2000 US elections)

Insincere Manipulation

Suppose that $X = \{x, y, z\}$ where x is the status quo, which is chosen unless there is a plurality winner.

1	1	1	1	1
y	x	z	z	
z	y	x	y	
x	z	y	x	

Winner: z

Insincere Manipulation

Suppose that $X = \{x, y, z\}$ where x is the status quo, which is chosen unless there is a plurality winner.

1	1	1	1
y	x	z	z
z	y	x	y
x	z	y	x

Winner: z

1	1	1	1
y	y	z	z
z	x	x	y
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Winner: x

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1	1	1	1		1
y	x	z	y		
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Winner: y

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1	1	1	1
y	x	z	y
z	y	x	z
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Winner: y

1	1	1	1
y	x	x	z
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Let $\mathbf{P} = (P_1, \ldots, P_n) \in L(X)$ be a profile and $S \subseteq N$ a group of individuals. The group S can **manipulate sincerely** iff there exists a profile $\mathbf{P}' = (P'_1, \ldots, P'_n)$ and a candidate A such that

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1. $P_j = P'_j$ for all $j \notin S$
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3. $F(\mathbf{P}') = A$ and $A \; P_i \; F(\mathbf{P})$ for all $i \in S$

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3. $F(\mathbf{P}') = A$ and $A\ P_i\ F(\mathbf{P})$ for all $i \in S$
4. for all profiles $\mathbf{P}''$ such that $\mathbf{P}''_j = P_j$ for all $j \notin S$: if $F(\mathbf{P}'') \neq A$, then $A\ P_i\ F(\mathbf{P}'')$ for some $i \in S$

A social choice function F is immune to insincere manipulation if for each group of voters S and profiles $\mathbf{P}$, if S has a possibility to manipulate given $\mathbf{P}$, then S can manipulate sincerely.

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A social choice function F satisfies **top-monotonicity** iff for all profiles $\mathbf{P} = (P_1, \ldots, P_n)$, $\mathbf{P}' = (P'_1, \ldots, P'_n)$ and all i: if $F(\mathbf{P})\ P'_i\ B$ for all $B \in X - \{F(\mathbf{P})\}$ whenever $P_i \neq P'_i$, then $F(\mathbf{P}) = F(\mathbf{P}')$.

Theorem (Dowding and van Hees). A social choice function F is immune to insincere manipulation if, and only if, it satisfies top-monotonicity.

Satterthwaite's Argument for Strategy-Proofness

1. Inequality of skills: the possibility of manipulation gives some voters an unfair advantage over others.

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4. Non-transparency of representatives' preferences: Manipulation blurs the voting records of politicians.

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3. Non-transparency of voters' preferences: The possibility of manipulation gives voters an incentive to hide their preferences.

4. Non-transparency of representatives' preferences: Manipulation blurs the voting records of politicians.

5. Randomness: Manipulation introduces an element of randomness into the voting process.

“The very fact that politics is a game, where the strategies include argument, persuasion, the understanding of the position of oneself and others, entail strategic interaction. In this regard strategic manipulation can be recognized as a virtue rather than a vice. The possibility of manipulation gives incentives for citizens to learn more about those around them, the political situation and the democratic process itself.” (pg. 14, Dowding and van Hees)

Manipulation of an election by an individual i requires

1. Knowledge of one's own preference
2. Knowledge of the other voters' preferences (knowledge of how everyone else is going to vote)
3. Knowledge of how the voting mechanism works

Will voters manipulate under perfect information?

Experimental setup: 3 players who have to vote on 4 outcomes using Borda count. Monetary payoffs set up to induce a specific ordering. One of the players is given full information about the other's preferences.

S. Kube and C. Puppe. *(When and how) do voters try to manipulate? Experimental evidence from Borda elections*. Public Choice, 139, pgs. 39-52, 2009.

Will voters manipulate under perfect information?

Result 1: Despite the possibility of successful manipulation, a non-negligable fraction reported their preferences truthfully. Yet, informed players tended to vote sincerely less often than uninformed voters.

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Result 2: The behavior of informed subjects is not affected by inequality considers, but strongly affected by the degree of uncertainty.

Result 3: Under uncertainty, rather than trying to bring about their best alternative, the majority of informed subjects show satisficing behavior: They try to secure themselves at least their second-best alternative.

S. Kube and C. Puppe. *(When and how) do voters try to manipulate? Experimental evidence from Borda elections*. Public Choice, 139, pgs. 39-52, 2009.

Will voters manipulate if they have no information?

Domination Manipulation

Let X be a set of candidates, $O(X)$ the set of preferences over X and $I \subseteq O(X)^n$ a set of profiles (an **information set**).

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Let X be a set of candidates, $O(X)$ the set of preferences over X and $I \subseteq O(X)^n$ a set of profiles (an **information set**).
A vote R **dominates** another vote Q for voter i (with a true preference ordering $\succeq_i$) with respect to I if:

for all profiles $\mathbf{R} = (R_1, \ldots, R_i, \ldots, R_n) \in I$:

$$F((R_1, \ldots, R, \ldots, R_n)) \succeq_i F(R_1, \ldots, Q, \ldots, R_n)$$

and there is some profile $\mathbf{R}' = (R'_1, \ldots, R'_i, \ldots, R'_n) \in I$ such that

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A social choice function is **immune to domination manipulation** if there is no vote that dominates a voter's "true" preferences.

Theorem (Conitzer, Walsh, and Xia).

- When the manipulator has no information, any Condorcet consistent voting rule is immune to dominating manipulation.
- When the manipulator has no information, Borda count is immune to dominating manipulation
- When the manipulator has no information and $n \geq 6(m-2)$, any positional scoring rule is immune to dominating manipulation.

V. Conitzer, T. Walsh and L. Xia. *Dominating manipulations in voting with partial information*. AAAI, 2011.

Uncertainty About the Profiles

How should we represent the a voter's uncertainty about how the other voters' preferences?

- Quantitative Information: Probability measure over space of profiles
- Qualitative Information: Information partition on the space of profiles

A poll information function: $p : O(X)^n \to \mathcal{I}$, where $\mathcal{I}$ is some property of the profiles. For example, A poll information function: $p : O(X)^n \to \mathcal{I}$, where $\mathcal{I}$ is some property of the profiles. For example,

- Profile: p is the identity function
- Ballot: p returns a vector recording how often each ballot occurs in the input profile
- (Weighted) Majority Graph: p returns the weighted majority graph
- Score: p returns the score for each candidate
- Rank: p returns the ranking of each candidate
- Winner: p returns the winner
- Zero: p returns 0

Let $p : O(X)^n \rightarrow \mathcal{I}$ be a poll information function and $\mathbf{R}$ a profile. The information cell generated by $\mathbf{R}$ is:

$$I_i(\mathbf{R}) = \{\mathbf{R}' \in O(X)^n \mid R'_i = R_i \text{ and } p(\mathbf{R}) = p(\mathbf{R}')\}$$

Strategic Voting and Epistemic Logic

H. van Ditmarsch, J. Lang and A. Saffidine. *Strategic voting and the logic of knowledge*. TARK, 2013.

S. Chopra, E. Pacuit and R. Parikh. *Knowledge-theoretic Properties of Strategic Voting*. JELIA 2004.

Iterated Voting

Allow voters the opportunity to *change* their votes in response to certain “poll” information (cf. J. Fishkin on *deliberative democracy*)

S. Chopra, E. Pacuit and R. Parikh. *Knowledge-theoretic Properties of Strategic Voting*. JELIA 2004.

A. Reijngoud and U. Endriss. *Voter Response to Iterated Poll Information*. AAMAS, 2012.

O. Lev and J. Rosenschein. *Convergence of Iterative Voting*. AAMAS, 2012.

Example I

The following example is due to [Brams & Fishburn]

$$
\begin{aligned}
P_A^* &= o_1 > o_3 > o_2 \\
P_B^* &= o_2 > o_3 > o_1 \\
P_C^* &= o_3 > o_1 > o_2
\end{aligned}
$$

Size	Group	I	II
4	A	$\mathbf{o_1}$	$\mathbf{o_1}$
3	B	o_2	o_2
2	C	o_3	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. o' is one of the top two candidates as indicated by a poll
2. o' is preferred to the other top candidate

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Example II

$$
\begin{aligned}
P_A^* &= (o_1, o_4, o_2, o_3) \\
P_B^* &= (o_2, o_1, o_3, o_4) \\
P_C^* &= (o_3, o_2, o_4, o_1) \\
P_D^* &= (o_4, o_1, o_2, o_3) \\
P_E^* &= (o_3, o_1, o_2, o_4)
\end{aligned}
$$

Size	Group	I	II	III	IV
40	A	$\mathbf{o_1}$	o_1	o_4	$\mathbf{o_1}$
30	B	o_2	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
15	C	o_3	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
8	D	o_4	o_4	o_1	o_4
7	E	o_3	o_3	o_1	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. i prefers o' to o, and
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30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

O. Lev and J. Rosenschein. *Convergence of Iterative Voting*. AAMAS, 2012.

Theorem (Lev and Rosenschein). An iterative scoring rule game with a deterministic tie-breaking rule that use best-response moves and start from a truth state, does not always converge.

Theorem (Lev and Rosenschein). An iterative Borda game with every type of tie-breaking rule that use a best response moves and start from a truthful state does not always converge.

Theorem (Lev and Rosenschein). An iterative k-approval game, when $k \geq 3$ with linear-ordered tie-breaking rule, that use best-response moves and start from a truthful state does not always converge.

A. Reijngoud and U. Endriss. *Voter Response to Iterated Poll Information*. AAMAS, 2012.

A **voter response strategy**:

- Truth-teller: always voter truthfully
- Strategist: Computes best response to poll information and uses (any) one of them.
- k-Pragmatist: Does not compute a best response, but rather moves her favorite of the currently k-highest ranked candidates to the top of her ballot.

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- Which properties of the social choice function are also satisfied by F^t? (Unanimity, Condorcet consistency)
- Run a simulation and measure the Condorcet efficiency of different voting rules.

Summary

- Which assumptions of the theorem or “crucial”? (non-resolute social choice functions, weak orders, partial orders, domain restrictions: single-peaked domains, etc.)

- What’s so *bad* about manipulation?

- How “hard” is it to manipulate an election?

- When will a voter misrepresent her preferences? What do the voters need to *know* (believe) about the other voters’ preferences?

- What is the effect of polling information on an election outcome?

Judgement Aggregation

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S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

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YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)
Proposition 2: *N* (7 - 6)
Proposition 3: *N* (7 - 6)

But there is no support for NNN!

Complete Reversal

YYYN	*YYNY*	*YNYY*	*NYYY*	*NNNN*
2	2	2	2	3

Outcome by majority vote

Proposition 1: Y (6 - 5)
Proposition 2: Y (6 - 6)
Proposition 3: Y (6 - 5)
Proposition 4: Y (6 - 5)

YYYY wins proposition-wise voting, but the "opposite" outcome *NNN* has the *most* overall support!

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"Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is." (pg. 234).

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

The Condorcet Jury Theorem

"Condorcet begins with the premise that the object of government is to make decisions that are in the best interest of society. This leads naturally to the question: what voting rules are most likely to yield good outcomes?"

(pg. 60)

H. P. Young. *Optimal Voting Rules*. The Journal of Economic Perspectives, 9:1, pgs. 51 - 64, 1995.

Suppose a group of people need to determine if some proposition A is true or false (e.g., is the defendant “guilty” or “innocent”).

Each voter i has a probability p_i of correctly identifying whether A is true or false.

Suppose that everyone is “competent”: for each voter i, $p_i > 1/2$.

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Suppose that everyone is "competent": for each voter i, $p_i > 1/2$.

Let F be a decision method. $\pi(F, \mathbf{p})$ is the probability of getting the answer correct, given the skills of each individual $\mathbf{p}$.

Expert rule

Suppose that there are three voters with the same probability $p > 1/2$ of getting the answer correct.

$$\pi(F^e, \mathbf{p}) = p$$

Majority Rule

Suppose that there are three voters with the same probability $p > 1/2$ of getting the answer correct.

$$\pi(F^m, \mathbf{p})$$

Majority Rule

Suppose that there are three voters with the same probability $p > 1/2$ of getting the answer correct.

$$\pi(F^m, \mathbf{p}) = p^3$$

The probability everyone is correct is p^3

Majority Rule

Suppose that there are three voters with the same probability $p > 1/2$ of getting the answer correct.

$$\pi(F^m, \mathbf{p}) = p^3 + 3p^2(1-p)$$

The probability everyone is correct is p^3

The probability that 1 and 2 are correct: $p^2(1-p)$
The probability that 2 and 3 are correct: $p^2(1-p)$
The probability that 1 and 3 are correct: $p^2(1-p)$

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Theorem. When there are three voters, each with a probability $p > 1/2$ of choosing correctly, then majority rule is preferred to the expert rule.

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Theorem. Assume $p_1 \geq p_2 > p_3 > 1/2$, then the simple majority rule is preferred to the expert rule.

What is the probability that at least m voters are correct? :

$$\sum_{h=m}^{n} \binom{n}{h} * p^h * (1-p)^{n-h}$$

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What is the probability that at least 2 people are correct in a population of 3 voters?

$$\binom{3}{2} * (2/3)^2 * 1/3^1 + \binom{3}{3} 2/3^3 * 1/3^0$$

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$$\binom{3}{2} * (2/3)^2 * 1/3^1 + \binom{3}{3} 2/3^3 * 1/3^0$$

$$= 3 * 4/27 + 1 * 8/27$$

$$\approx 0.74$$

What is the probability that at least m voters are correct? :

$$\sum_{h=m}^{n} \binom{n}{h} * p^h * (1-p)^{n-h}$$

What is the probability that at least 3 people are correct in a population of 5 voters?

$$\binom{5}{3} * (2/3)^3 * 1/3^2 + \binom{5}{4}(2/3)^4 * 1/3^1 + \binom{5}{5}2/3^5 * 1/3^0$$

$$= 10 * 8/243 + 5 * 16/243 + 1 * 32/243$$

$$\approx 0.79$$

Condorcet Jury Theorem

Suppose that the “state of the world” $\mathbf{x}$ takes values 0 and 1

R_i is the event that voter i reports correctly.

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R_i is the event that voter i reports correctly.

Independence The reports of the voters are independent conditional on the state of the world: $R_1, R_2, \ldots$ are *independent conditional*on $\mathbf{x}$

Competence: For each voter, the probability that the reports correctly is greater than 1/2: for each $x \in \{0, 1\}$, $p(R_i \mid \mathbf{x} = x) > \frac{1}{2}$ and

Condorcet Jury Theorem

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Condorcet Jury Theorem. Suppose Independence and Competence. As the group size increases, the probability that majority opinion is correct (i) increases and (ii) converges to one.

Literature

D. Austen-Smith and J. Banks. *Aggregation, Rationality and the Condorcet Jury Theorem*. The American Political Science Review, 90:1, pgs. 34 - 45, 1996.

D. Estlund. *Opinion Leaders, Independence and Condorcet's Jury Theorem*. Theory and Decision, 36, pgs. 131 - 162, 1994.

F. Dietrich. *The premises of Condorcet's Jury Theorem are not simultaneously justified*. Episteme,5:1, pgs. 56 - 73, 2008.

S. Nitzan. *Collective Preference and Choice (Part III)*. Cambridge University Press, 2010.

Judgement Aggregation

Suppose that three experts *independently* formed opinions about three propositions. For example,

1. p: "Carbon dioxide emissions are above the threshold x"
2. $p \to q$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"
3. q: "There will be global warming"

	p	$p \to q$	q
Expert 1			
Expert 2			
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	
Expert 2			
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2			
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group			

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True		

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	False

p: a valid contract was in place
q: there was a breach of contract
r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

An employee-owned bakery must decide whether to buy a pizza oven (P) or a fridge to freeze their outstanding Tiramisu (F). The pizza oven and the fridge cannot be in the same room. So they also need to decide whether to rent an extra room in the back (R). They all agree that they will rent the room if they decide to buy both the pizza oven and the fridge: $((P \wedge F) \rightarrow R)$, but they are contemplating renting the room regardless of the outcome of the vote on the appliances.

F. Cariani. *Judgement Aggregation*. Philosophy Compass, 6, 1, pgs. 22 - 32.

P, F are reasons for R

$\neg P, \neg F$ are not reasons for $\neg R$

$\neg R, P$ are reasons for $\neg F$

Fabrizio Cariani, Marc Pauly, and Josh Snyder. *Decision Framing in Judgment Aggregation*. Synthese, Volume 163, Issue 1, pp 124, 2008.

In 1991, the German parliament staged a debate on whether the parliament should move from Bonn to Berlin

Among the motions considered were A (the parliament should move to Berlin), and B (the seat of government should move to Berlin)

	A	B	$A \wedge B$
1	T	T	T
2	T	F	F
3	F	T	F

In 1991, the German parliament staged a debate on whether the parliament should move from Bonn to Berlin

Among the motions considered were A (the parliament should move to Berlin), and B (the seat of government should move to Berlin)

Should the parliament and the government should not be geographically separated?

	A	B	$A \wedge B$	$A \leftrightarrow B$
1	T	T	T	T
2	T	F	F	F
3	F	T	F	F

Another decision frame might have looked equally good. In the new frame, the basic motions they consider are whether to move the parliament (A'), and whether parliament and government should be in the same city (B').

	A	B	$A \wedge B$	$A \leftrightarrow B$
1	T	T	T	T
2	T	F	F	F
3	F	T	F	F

	A'	B'	$A' \wedge B'$
1	T	T	T
2	T	F	F
3	F	F	F

The Propositions

Propositions: Let $\mathcal{L}$ be a logical language (called **propositions** in the literature) with the usual boolean connectives.

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Consistency: The standard notion of logical consistency.

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Consistency: The standard notion of logical consistency.

Aside: We actually need

1. $\{p, \neg p\}$ are inconsistent
2. all subsets of a consistent set are consistent
3. $\emptyset$ is consistent and each $S \subseteq \mathcal{L}$ has a consistent maximal extension (not needed in all cases)

The Agenda

Definition The **agenda** is a non-empty set $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are made (note: X is a union of proposition-negation pairs $\{p, \neg p\}$).

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Example: In the discursive dilemma: $X = \{p, \neg p, q, \neg q, p \rightarrow q, \neg(p \rightarrow q)\}$.

The Judgement Sets

Definition: Given an agenda X, each individual i's judgement set is a subset $A_i \subseteq X$.

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Definition: Given an agenda X, each individual i's judgement set is a subset $A_i \subseteq X$.

Rationality Assumptions:

1. A_i is **consistent**
2. A_i is **complete**, if for each $p \in X$, either $p \in A_i$ or $\neg p \in A_i$

Aggregation Rules

Let X be an agenda, $N = \{1, \ldots, n\}$ a set of voters, a **profile** is a tuple $(A_i, \ldots, A_n)$ where each A_i is a judgement set. An **aggregation function** is a map from profiles to judgment sets. I.e., $F(A_1, \ldots, A_n)$ is a judgement set.

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Examples:

- **Propositionwise majority voting**: for each $(A_1, \ldots, A_n)$,

$$F(A_1, \ldots, A_n) = \{p \in X \mid |\{i \mid p \in A_i\}| \geq |\{i \mid p \notin A_i\}|\}$$

- **Dictator of** i: $F(A_1, \ldots, A_n) = A_i$
- **Reverse Dictator of** i: $F(A_1, \ldots, A_n) = \{\neg p \mid p \in A_i\}$

Input

Universal Domain: The domain of F is the set of all possible profiles of consistent and complete judgement sets.

Output

Collective Rationality: F generates consistent and complete collective judgment sets.

Anonymity: For all profiles $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = F(A_{\pi(1)}, \ldots, A_{\pi(n)})$ where π is a permutation of the voters.

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Unanimity: For all profiles $(A_1, \ldots, A_n)$ if $p \in A_i$ for each i then $p \in F(A_1, \ldots, A_n)$

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Monotonicity: For any $p \in X$ and all $(A_1, \ldots A_i, \ldots, A_n)$ and $(A_1, \ldots, A_i^*, \ldots, A_n)$ in the domain of F,

$$\text{if } [p \notin A_i, p \in A_i^* \text{ and } p \in F(A_1, \ldots, A_i, \ldots A_n)]$$
$$\text{then } [p \in F(A_1, \ldots, A_i^*, \ldots A_n)].$$

Systematicity: For any $p, q \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N, p \in A_i \text{ iff } q \in A_i^*]$$
$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } q \in F(A_1^*, \ldots A_n^*) \text{]}.$$

Systematicity: For any $p, q \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

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$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } q \in F(A_1^*, \ldots A_n^*) \text{]}.$$

- independence
- neutrality

Independence: For any $p \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N, p \in A_i \text{ iff } p \in A_i^*]$$
$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } p \in F(A_1^*, \ldots A_n^*)\ ].$$

Non-dictatorship: There exists no $i \in N$ such that, for any profile $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = A_i$

Baseline Result

Theorem (List and Pettit, 2001) If $X \subseteq \{a, b, a \wedge b\}$, there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.

Agenda Richness

Whether or not judgment aggregation gives rise to serious impossibility results depends on how the propositions in the agenda are *interconnected*.

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Whether or not judgment aggregation gives rise to serious impossibility results depends on how the propositions in the agenda are *interconnected*.

Definition A set $Y \subseteq \mathcal{L}$ is **minimally inconsistent** if it is inconsistent and every proper subset $X \subsetneq Y$ is consistent.

Agenda Richness

Definition An agenda X is **minimally connected** if

1. (*non-simple*) it has a minimal inconsistent subset $Y \subseteq X$ with $|Y| \geq 3$
2. (*even-number-negatable*) it has a minimal inconsistent subset $Y \subseteq X$ such that

$$Y - Z \cup \{\neg z \mid z \in Z\} \text{ is consistent}$$

   for some subset $Z \subseteq Y$ of even size.

Impossibility Theorems

Theorem (Dietrich and List, 2007) If (and only if) an agenda is non-simple and even-number negatable, every aggregation rule satisfying universal domain, collective rationality, systematicity and unanimity is a dictatorship (or inverse dictatorship).

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Theorem (Nehring and Puppe, 2002) If (and only if) an agenda is non-simple, every aggregation rule satisfying universal domain, collective rationality, systematicity, unanimity, and monotonicity is a dictatorship.

Characterization Result

$p \in X$ **conditionally entails** $q \in X$, written $p \vdash^* q$ provided there is a subset $Y \subseteq X$ consistent with each of p and $\neg q$ such that $\{p\} \cup Y \vdash q$.

Totally Blocked: X is totally blocked if for any $p, q \in X$ there exists $p_1, \ldots, p_k \in X$ such that

$$p = p_1 \vdash^* p_2 \vdash^* \cdots \vdash^* p_k = q$$

Characterization Result

Theorem (Dietrich and List, 2007, Dokow Holzman 2010) If (and only if) an agenda is totally blocked and even-number negatable, every aggregation rule satisfying universal domain, collective rationality, independence and unanimity is a dictatorship.

Theorem (Nehring and Puppe, 2002, 2010) If (and only if) an agenda is totally blocked, every aggregation rule satisfying universal domain, collective rationality, independence, unanimity, and monotonicity is a dictatorship.

$C \subseteq N$ is **winning for** p if for all profiles $\mathbf{A} = (A_1, \ldots, A_n)$, if $p \in A_i$ for all $i \in C$ and $p \notin A_j$ for all $j \notin C$, then $p \in F(\mathbf{A})$

$\mathcal{C}_p = \{C \mid C \text{ is winning for } p\}$

Proof Sketch

1. If the agenda is totally blocked, then $C_p = C_q$ for all p, q. Let $C = C_p$ for some p (hence for all p).

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2. If $C \in \mathcal{C}$ and $C \subseteq C'$, then $C' \in \mathcal{C}$.

Proof Sketch

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2. If $C \in \mathcal{C}$ and $C \subseteq C'$, then $C' \in \mathcal{C}$.

3. If $C_1, C_2 \in \mathcal{C}$, then $C_1 \cap C_2 \in \mathcal{C}$.

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2. If $C \in \mathcal{C}$ and $C \subseteq C'$, then $C' \in \mathcal{C}$.
3. If $C_1, C_2 \in \mathcal{C}$, then $C_1 \cap C_2 \in \mathcal{C}$.
4. $N \in \mathcal{C}$.
5. For all $C \subseteq N$, either $C \in \mathcal{C}$ or $\overline{C} \in \mathcal{C}$.

Proof Sketch

1. If the agenda is totally blocked, then $\mathcal{C}_p = \mathcal{C}_q$ for all p, q. Let $\mathcal{C} = \mathcal{C}_p$ for some p (hence for all p).

2. If $C \in \mathcal{C}$ and $C \subseteq C'$, then $C' \in \mathcal{C}$.

3. If $C_1, C_2 \in \mathcal{C}$, then $C_1 \cap C_2 \in \mathcal{C}$.

4. $N \in \mathcal{C}$.

5. For all $C \subseteq N$, either $C \in \mathcal{C}$ or $\overline{C} \in \mathcal{C}$.

6. There is an $i \in N$ such that $\{i\} \in \mathcal{C}$.

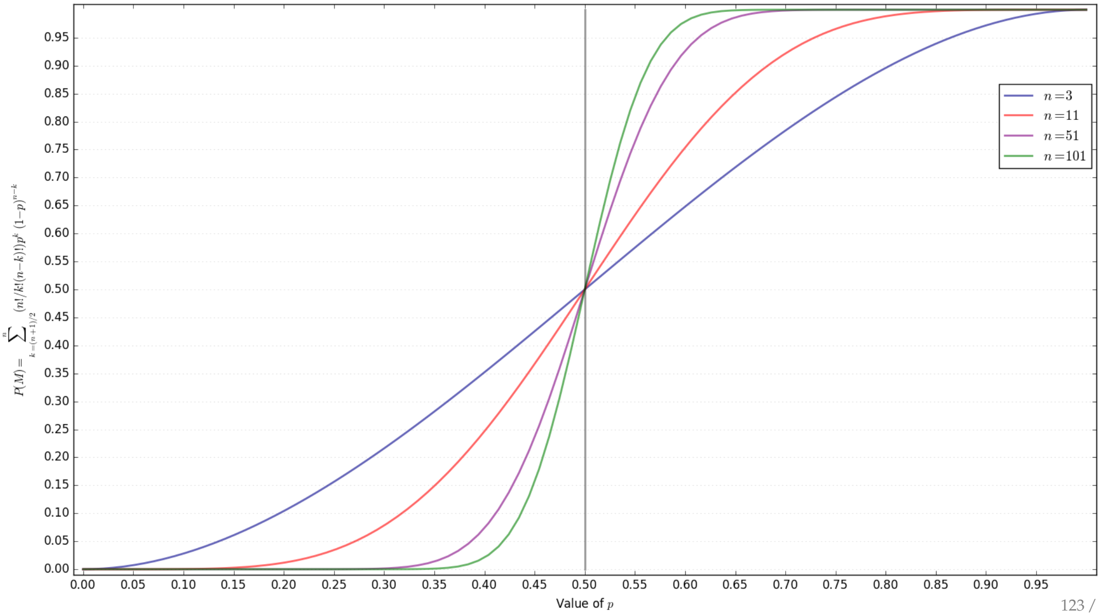
Many Variants!

C. List. *The theory of judgment aggregation: An introductory review*. Synthese 187(1), pgs. 179-207, 2012.

D. Grossi and G. Pigozzi. *Judgement Aggregation: A Primer*. Morgan & Claypol, 2014.

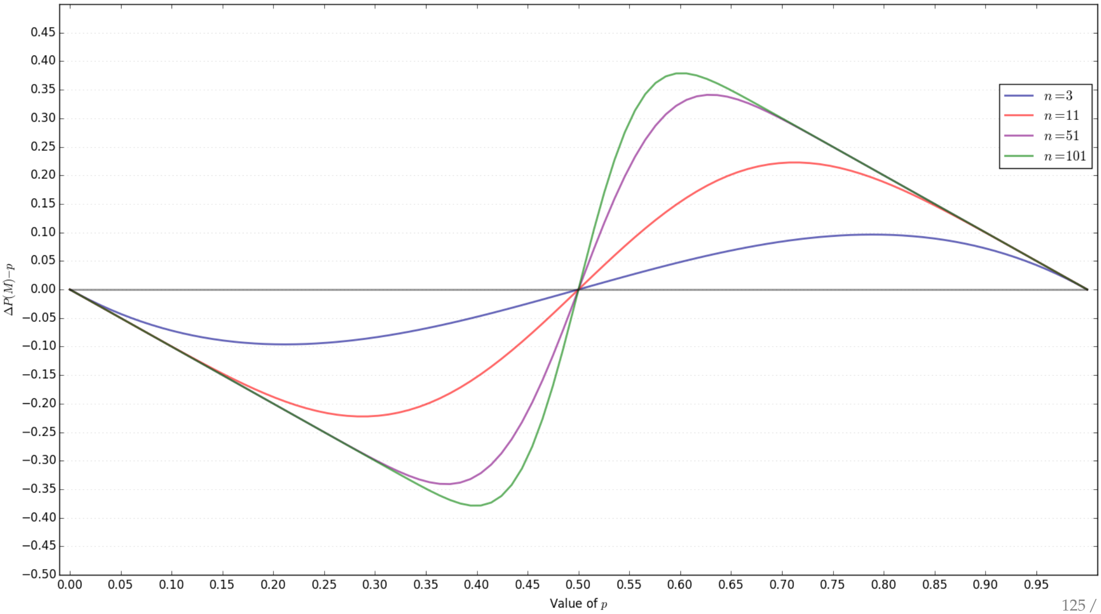
L. Bovens and W. Rabinowicz. *Voting Procedures for Complex Collective Decisions. An Epistemic Perspective*. Ratio Juris, 17:2, pp. 241-258, 2004.

$$P(M) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$



$P(M) = \sum_{k=(n+1)/2}^{n} (n!/k!(n-k)!) p^k (1-p)^{n-k}$
Value of p
$n=3$
$n=11$
$n=51$
$n=101$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95

$$\Delta = P(M) - p$$



$n=3$
$n=11$
$n=51$
$n=101$
$\Delta P(M)-p$
Value of p

	S	F	$D \leftrightarrow (F \wedge S)$
C1	T	T	T
C2	T	F	F
C3	F	T	F
C4	F	F	F

	S	F	$D \leftrightarrow (F \wedge S)$
C1	T	T	T
$C2$	T	F	F
$C3$	F	T	F
$C4$	F	F	F

$$P(C1) = q^2$$

$$P(C2) = P(C3) = q(1 - q)$$

$$P(C4) = (1 - q)^2$$

$$P(V \mid C1) = p^2$$

$$P(V \mid C2) = p^2 + p(1 - p) + (1 - p)^2$$

$$P(V \mid C4) = p^2 + 2p(1 - p)$$

$$P(V) = \sum_{i=1}^{4} P(V \mid Ci)P(Ci)$$

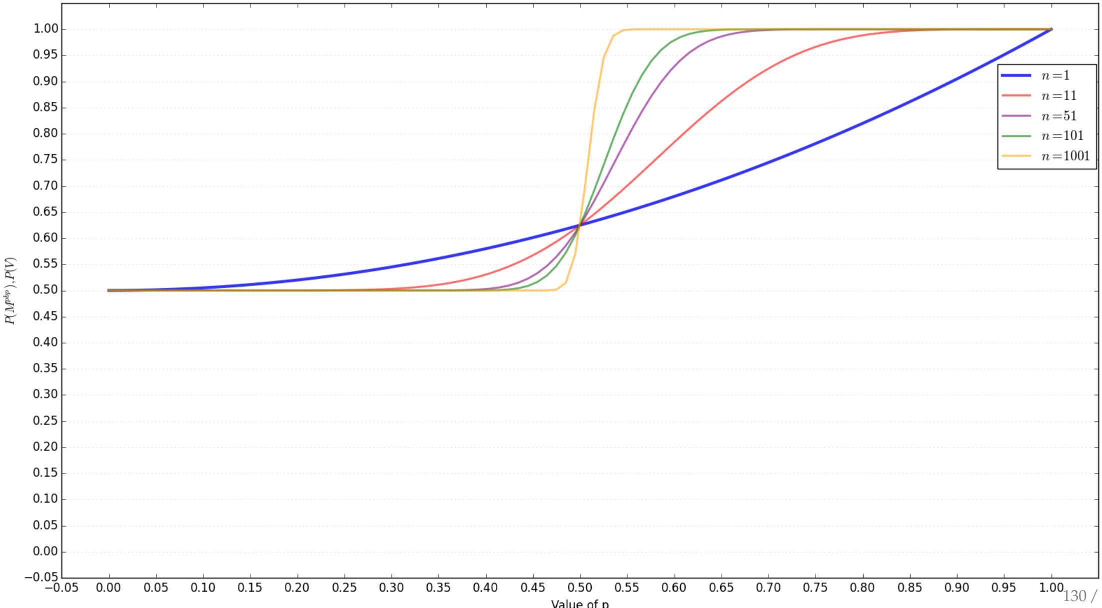
$$P(M^{pbp} \mid C1) = P(M)^2$$

$$P(M^{pbp} \mid C2) = P(M^{pbp} \mid C3) = P(M)^2 + P(M)(1 - P(M)) + (1 - P(M))^2$$

$$P(M^{pbp} \mid C4) = P(M)^2 + 2P(M)(1 - P(M))$$

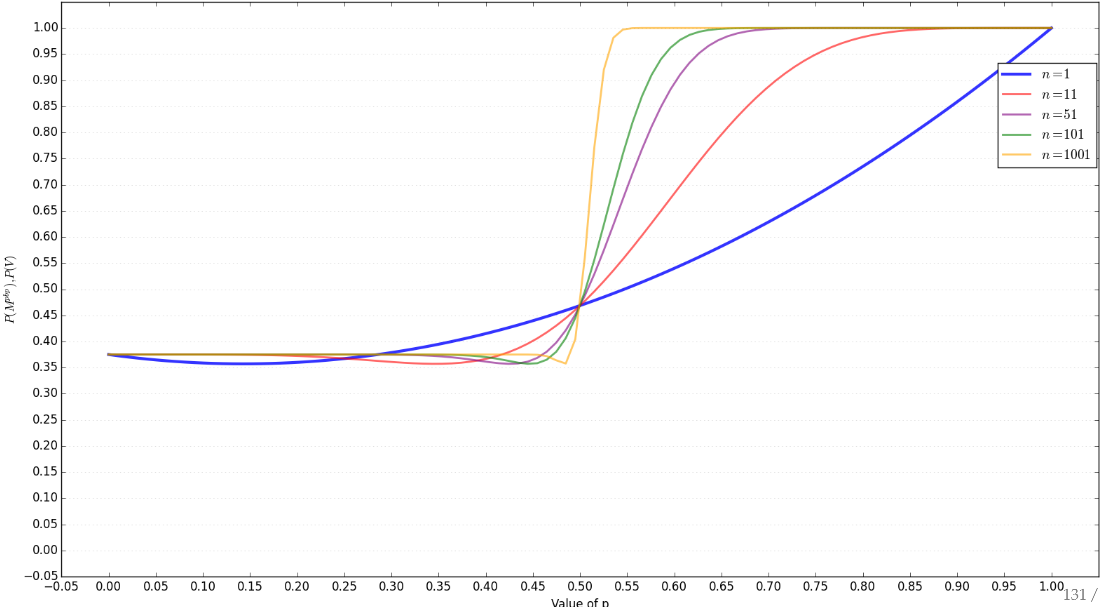
$$P(M^{pbp}) = \sum_{i=1}^{4} P(M^{pbp} \mid Ci)P(Ci)$$

$q = 0.5$



1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.05
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
$P(M^{pbp}).P(V)$
Value of p
$n = 1$
$n = 11$
$n = 51$
$n = 101$
$n = 1001$

$q = 0.75$

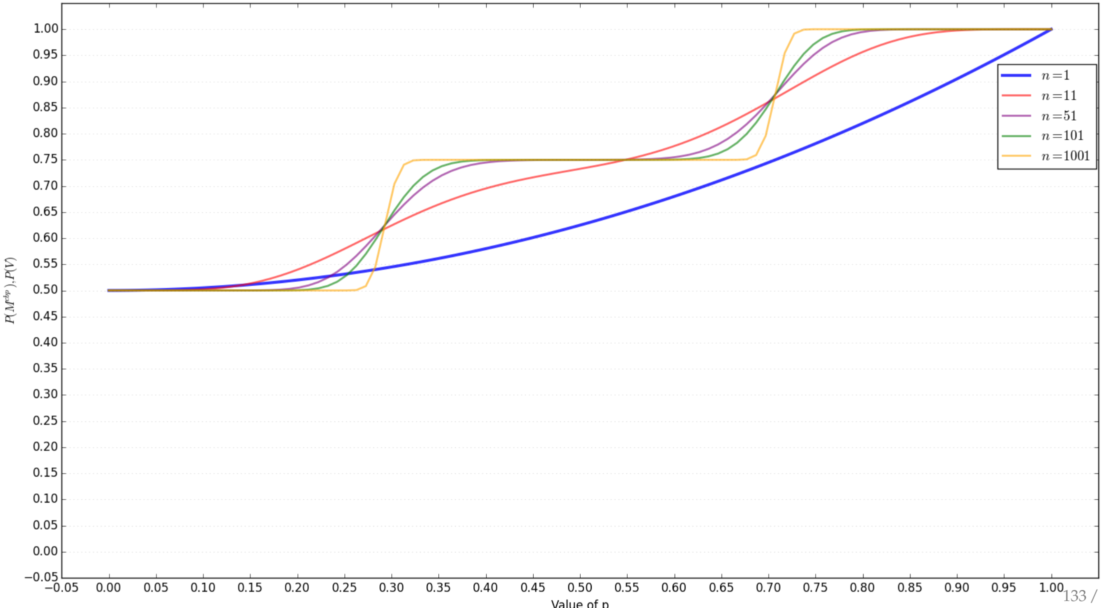


$n = 1$
$n = 11$
$n = 51$
$n = 101$
$n = 1001$
$P(M^{pbp}), P(V)$
Value of p
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.05 0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00

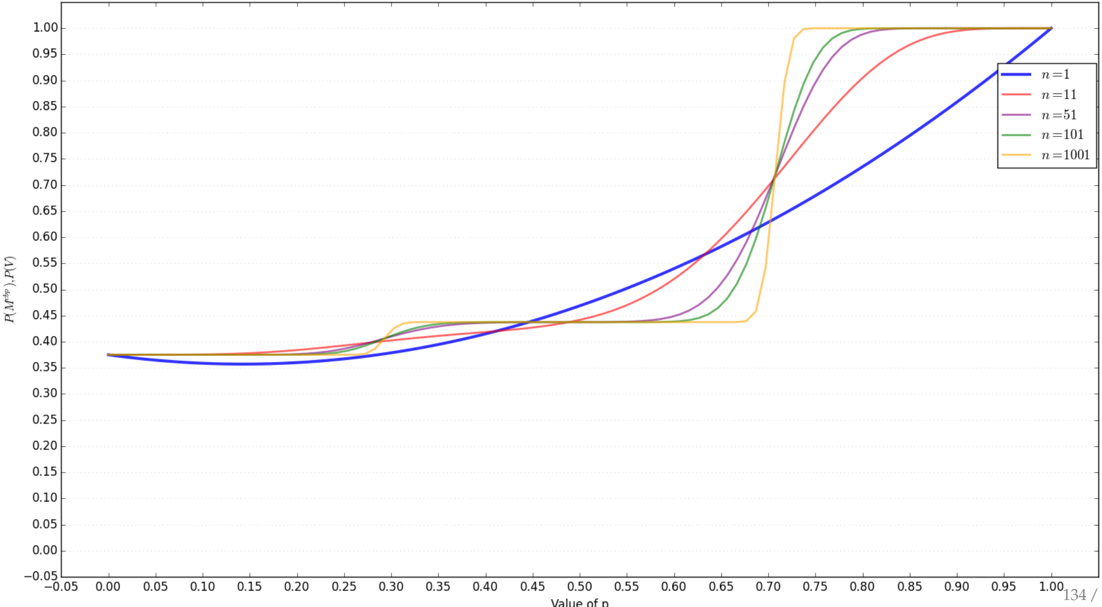
$$P(M^{cbp} \mid Ci) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} P(V \mid Ci)^k (1 - P(V \mid Ci))^{n-k}$$

$$P(M^{cbp}) = \sum_{i=1}^{4} P(M^{cbp} \mid Ci) P(Ci)$$

$q = 0.5$



$q = 0.75$



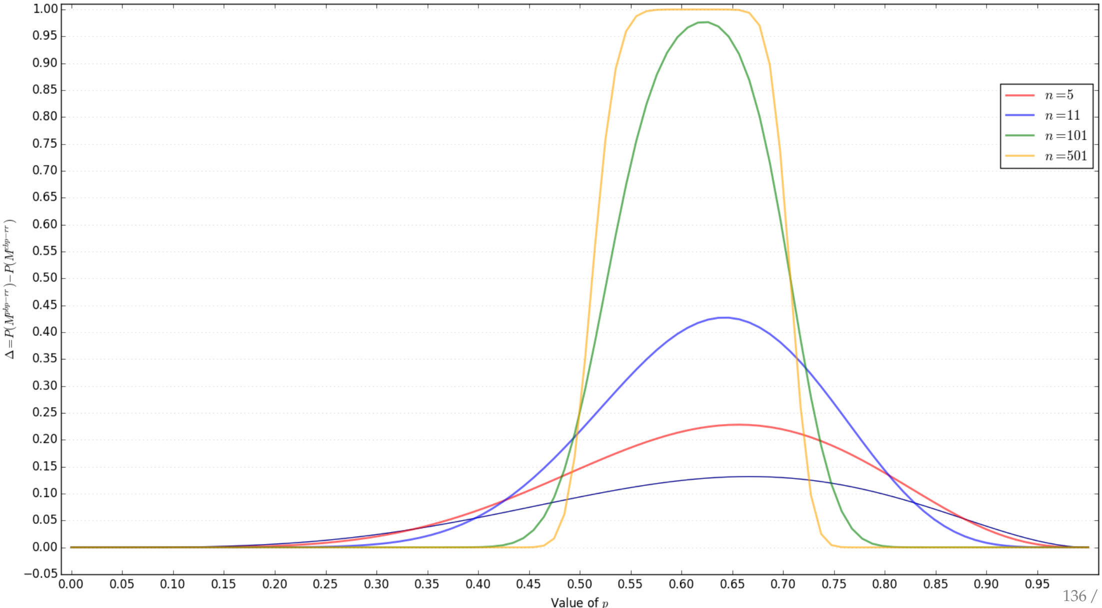
$$P(M^{pbp}) = \sum_{i=1}^{4} P(M^{pbp} \mid Ci)P(Ci)$$

$$P(M^{pbp-rr}) = P(M)^2$$

$$P(M^{cbp}) = \sum_{i=1}^{4} P(M^{cbp} \mid Ci)P(Ci)$$

$$P(M^{cbp-rr}) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} p^2(1-p^2)^{n-k}$$

$q = 0.5$



$n = 5$
$n = 11$
$n = 101$
$n = 501$
$\Delta = P(M^{pbp-rr}) - P(M^{cbp-rr})$
Value of p
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95

Plan

- Introduction, Background, Voting Theory, May's Theorem, Arrow's Theorem
- Social Choice Theory: Variants of Arrow's Theorem, Weakening Arrow's Conditions (Domain Conditions), Harsanyi's Theorem, Characterizing Voting Methods
- Strategizing (Gibbard-Satterthwaite Theorem) and Iterative Voting/ Introduction to Judgement Aggregation
- Aggregating Judgements (linear pooling, wisdom of the crowds, prediction markets), Probabilistic Social Choice.
- Logics for Social Choice Theory (Modal Logic, Dependence/Independence Logic, First Order Logic)